Towards a Formalization of Skepticism in Extension-based Argumentation Semantics

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Abstract. This paper provides a preliminary investigation towards the definition of a general framework for the comparison of extension-based argumentation semantics with respect to the notion of skepticism. We identify seven justification states for arguments and define two alternative skepticism relations between semantics, which induce a partial order on the justification states, reflecting the relevant levels of commitment.

1 INTRODUCTION

According to Webster's dictionary a skeptical person is one "not easily persuaded or convinced". It is a common experience that very different degrees of "easiness of persuasion" can be met in everyday reasoning, ranging from credulous to extremely conservative attitudes: even when sharing the same initial information and the same reasoning steps, two people may reach different convictions about some topics, according to their propension towards skepticism. In fact, due to the uncertainty typically affecting both premises and rules of inference, reasoning conclusions may conflict each other and different approaches to solve these conflicts may be adopted.

These phenomena are captured by argumentation theory, where the reasoning activity is modeled as the process of constructing arguments for propositions, which are then evaluated, on the basis of their conflict relationships, according to a specified semantics. The seminal work by Dung [6] provides an abstract framework which represents a unifying view of several alternative semantics. Dung's theory is able to encompass a variety of existing proposals in the areas of nonmonotonic reasoning, ranging from logic programming to defeasible logics and game theory. Given an argumentation framework, the fundamental idea is that of identifying a set of *extensions*, each one representing a conflict-free set of arguments deemed to be collectively acceptable. Defining a specific *argumentation semantics* amounts to specifying the criteria for deriving a set of extensions from an argumentation framework. On the basis of such set of extensions, each argument can then be assigned a justification status; in particular, an argument is considered as justified if it belongs to all extensions. Within this framework, alternative literature proposals differ both in the underlying notion of extension and in the way the justification status of arguments is conceived.

In order to provide a formal counterpart to the different natural attitudes mentioned above, it is required to compare different semantics with respect to the property of *skepticism*. Intuitively, a semantics is more skeptical than another if it makes less committed choices about the justification status of the arguments. Roughly, the most skeptical conceivable semantics does not make any commitment on any argument, leaving all of them undecided, while a semantics is not skeptical at all if it takes a definite position about every argument in any argumentation framework. A comparison of skepticism between semantics can be based on relationships either among extensions or among justification states of arguments. However, these two alternative perspectives are clearly related, since justification states directly depend on the set of extensions prescribed by the semantics. It is also worth noting that two semantics may not be comparable with respect to skepticism, for instance in case they do not agree about some definitely committed choices: if an argument is justified in a semantics and rejected in another, comparing them about skepticism is not meaningful. As a consequence, the relation of skepticism is in general a partial order.

The issue of skepticism has been considered so far in the literature only in the case of specific proposals, while a reference framework able to support an analysis of skepticism at a more general level is still lacking. In particular, a relatively limited attention has been paid to the problem of systematically characterizing the justification states of arguments, which is important not only with respect to skepticism analysis but, more generally, for a better understanding of the notions of acceptance and rejection which may admit a variety of intermediate levels.

This paper aims at carrying out a joint investigation about the two main issues mentioned above. First of all, we introduce a novel classification encompassing seven justification states for arguments, we define and analyze two general relations of skepticism between semantics and we show how these relations induce two different partial orders (actually, meet semi-lattices) on the argument states. Since the underlying basic framework adopted for our analysis, namely Dung's theory, is very general and our proposal is applicable to any specific semantics, we believe that the abstract concepts presented in this paper may have a significant role both in stimulating further discussions on the general issue of skepticism in argumentation and in supporting useful characterizations of several applications of argumentation theory.

The proposal presented in this paper is only a first step towards the definition of a general approach to the notion of skepticism; research on several challenging issues is currently in progress.

2 BACKGROUND

The general theory proposed by Dung [6] is based on the primitive notion of *argumentation framework*:

Definition 1 *An argumentation framework is a pair* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$, *where* $\mathcal{A}$ *is a set, and* $\rightarrow \subseteq (\mathcal{A} \times \mathcal{A})$ *is a binary relation on* $\mathcal{A}$.

The idea is that arguments are simply conceived as the elements of the set $\mathcal{A}$, whose origin is not specified, and the interaction between

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them is modeled by the binary relation of attack $\rightarrow$.

In order to introduce different notions of extension, the fundamental concepts of acceptability and admissibility are defined[2]:

Definition 2 *Given an argumentation framework* $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$*:*

- *A set* $S \subseteq \mathcal{A}$ *is* conflict-free *iff* $\nexists \alpha, \beta \in S$ *such that* $\alpha \rightarrow \beta$*.*
- *An argument* $\alpha \in \mathcal{A}$ *is* acceptable *with respect to a set* $S \subseteq \mathcal{A}$ *iff* $\forall \beta \in \mathcal{A}$*, if* $\beta \rightarrow \alpha$ *then also* $S \rightarrow \beta$*.*
- *A set* $S \subseteq \mathcal{A}$ *is* admissible *iff* S *is conflict-free and each argument in* S *is acceptable with respect to* S*, i.e.* $\forall \beta \in \mathcal{A}$ *such that* $\beta \rightarrow S$ *we have that* $S \rightarrow \beta$*.*

On this basis, the notion of *complete extension*, introduced as a unifying concept underlying all of the proposed semantics, is defined as an admissible set $E \subseteq \mathcal{A}$ such that every argument $\alpha \in \mathcal{A}$ which is acceptable with respect to E belongs to E. Then, the two classical approaches to argumentation semantics can be introduced, namely the *grounded* and *preferred* semantics.

The grounded semantics adheres to the so-called *unique-status approach*, since for a given argumentation framework AF it always identifies a single extension $\mathrm{GE}_{\mathrm{AF}}$, called *grounded extension*, corresponding to the least (wrt. $\subseteq$) complete extension of AF. The set of arguments $\mathcal{A}$ can then be partitioned into:

- *undefeated arguments*, that belong to $\mathrm{GE}_{\mathrm{AF}}$ and are considered as justified;
- *defeated argument*, that are attacked by $\mathrm{GE}_{\mathrm{AF}}$ and are rejected;
- *provisionally defeated arguments*, that are neither included in $\mathrm{GE}_{\mathrm{AF}}$ nor attacked by it, reflecting in a sense a sort of undecided status.

Preferred semantics follows, instead, a multiple-status approach: the set of *preferred extensions*, denoted as $\mathcal{PE}_{\mathrm{AF}}$, is defined as the set of all maximal admissible sets, or equivalently of all maximal complete extensions. From these definitions, it follows that the grounded extension is included in all preferred extensions, therefore all arguments that are undefeated according to grounded semantics are also justified according to preferred semantics, and all arguments defeated according to grounded semantics are attacked by any preferred extension, and therefore they are not justified. Aside from this basic agreement, preferred semantics supports a finer discrimination of argument status in the cases concerning the so-called *floating arguments*. Considering for instance the argumentation framework AF_1 of Figure 1, it is easy to see that, according to grounded semantics, all arguments are provisionally defeated, while $\mathcal{PE}_{\mathrm{AF}_1} = \{\{\alpha, \delta\}, \{\beta, \delta\}\}$, therefore δ is included in all preferred extensions and is justified according to preferred semantics. In a sense, the state of δ, that is left uncommitted by grounded semantics, is assigned to justified by the preferred semantics.

This reasoning has been traditionally the basis for the comparison of grounded and preferred semantics with respect to skepticism: clearly, the former is more skeptical than the latter. Intuitively, in grounded semantics the state of provisionally defeated is less committed than the other ones, therefore, in presence of provisionally defeated arguments, preferred semantics can behave less skeptically by assigning a more committed (i.e. rejected or accepted) state to some of them. This suggests that, when comparing two generic semantics, it is necessary to characterize justification states according to a partial order reflecting the relevant levels of commitment: a semantics is less skeptical than another if it assigns to each argument a

[2] We extend the attack relation $\rightarrow$ as follows: given an argument α and a set of arguments S, $S \rightarrow \alpha$ iff $\exists \beta \in S : \beta \rightarrow \alpha$, $\alpha \rightarrow S$ iff $\exists \beta \in S : \alpha \rightarrow \beta$.

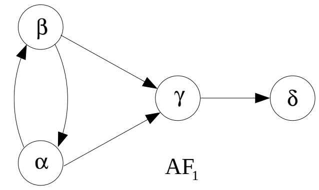

Figure 1. Floating arguments

state which, with respect to such order, is greater than (or equal to) that assigned by the more skeptical one.

The first step in this direction is therefore to identify the different possible justification states of an argument.

3 IDENTIFYING THE JUSTIFICATION STATES

In Dung's framework, basically three justification states for an argument are envisaged on the basis of its membership to extensions: an argument may belong to all extensions, to no extension or to some (not all) of them, i.e. to a strict subset of them. Recently, in [5] a more refined classification encompassing four states has been introduced. An argument can be:

1. *uni-accepted* if it belongs to all extensions;
2. *not-accepted* if it doesn't belong to any extension;
3. *cleanly-accepted* if it belongs to at least one extension and is not attacked by any extension;
4. *only-exi-accepted* if it belongs to at least one extension and is attacked by at least one extension.

While this classification has been defined to explore graduality in the so-called interaction-based valuation of arguments, it turns out that it is not adequate for the analysis of the concept of skepticism. In fact, considering any single status approach, such as the grounded semantics, only the first two states are applicable, since an argument can just be in or out with respect to the only existing extension. As a consequence, defeated and provisionally defeated arguments collapse in a unique state, thus preventing any ordering on the states of arguments and, at the same time, any skepticism comparison. A more systematic analysis is therefore needed.

As a starting point, we consider the relationship between an argument α and a particular extension E; three main situations can be envisaged, namely

- *in* E, if $\alpha \in E$;
- *definitely out from* E, if $\alpha \notin E \wedge E \rightarrow \alpha$;
- *provisionally out from* E, if $\alpha \notin E \wedge E \not\rightarrow \alpha$.

Taking into account now the existence of multiple extensions, one can consider that an argument can be in any of the above three states with respect to all, some or none of the extensions. This gives rise to 27 hypothetical combinations. It is however easy to see that some of them are impossible, for instance if an argument is in a given state with respect to all extensions this clearly excludes that it is in another state with respect to any extension. Directly applying this kind of considerations, seven possible *Justification States* emerge for an argument α with respect to a set of extensions $\mathcal{E}$:

JS1 $\forall E \in \mathcal{E}$, α is in E;
JS2 $\forall E \in \mathcal{E}$, α is definitely out from E;
JS3 $\forall E \in \mathcal{E}$, α is provisionally out from E;

JS4 $\exists E \in \mathcal{E}$ such that α is definitely out from E, $\exists E \in \mathcal{E}$ such that α is provisionally out from E, and $\nexists E \in \mathcal{E}$ such that α is in E;

JS5 $\exists E \in \mathcal{E}$ such that α is in E, $\exists E \in \mathcal{E}$ such that α is provisionally out from E, and $\nexists E \in \mathcal{E}$ such that α is definitely out from E;

JS6 $\exists E \in \mathcal{E}$ such that α is in E, $\exists E \in \mathcal{E}$ such that α is definitely out from E, and $\nexists E \in \mathcal{E}$ such that α is provisionally out from E;

JS7 $\exists E \in \mathcal{E}$ such that α is in E, $\exists E \in \mathcal{E}$ such that α is definitely out from E, and $\exists E \in \mathcal{E}$ such that α is provisionally out from E.

It is easy to see that if the semantics enforces a unique-status approach, i.e. $|\mathcal{E}| = 1$, then only $JS1$, $JS2$ and $JS3$ may hold. In case of the grounded semantics, i.e. $\mathcal{E} = \{\mathrm{GE_{AF}}\}$, they correspond to the status of undefeated, defeated and provisionally defeated, respectively.

A relevant question concerns the actual existence of each of the seven states, i.e. whether for each JSi an argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ and a semantics exist such that one argument $\alpha \in \mathcal{A}$ is in JSi. Actually, it is possible to provide an example for each state JSi in the context of preferred semantics. Considering AF_1 shown in Figure 1, it is easy to see that δ is in $JS1$, γ is in $JS2$, while both α and β are in $JS6$. As far as $JS3$ is concerned, it is sufficient to consider an argumentation framework consisting of a single odd-length cycle, which admits the empty set as the unique preferred extension assigning to all of the arguments the state $JS3$. As for $JS4$ and $JS5$, the argumentation framework AF_2 in Figure 2 admits as preferred extensions the sets $\{\phi_1, \beta\}$ and $\{\phi_2\}$, therefore α is in $JS4$ and β is in $JS5$. Finally, it can be seen that for the argumentation framework AF_3, shown in Figure 3, $\mathcal{PE}_{\mathrm{AF}_3}$ includes among others the sets $\{\phi_1, \phi_3\}$, $\{\phi_2, \phi_4, \gamma\}$ and $\{\phi_1, \phi_4, \beta\}$, therefore in particular β is in $JS7$. This proves the actual existence of all the seven states defined above.

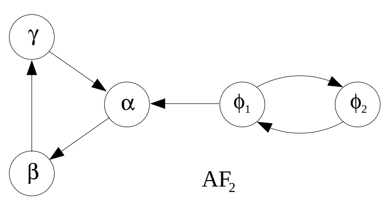

Figure 2. Example for $JS4$ and $JS5$.

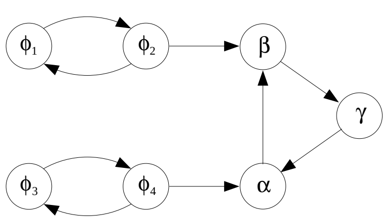

Figure 3. Example for $JS7$.

Since a direct comparison among these states with respect to the level of commitment is not straightforward, in order to figure out an ordering among the states we need to introduce a general relation of skepticism between sets of extensions.

4 FORMALIZING THE CONCEPT OF SKEPTICISM

In this paper a semantics S is identified with the set of extensions it prescribes for a given argumentation framework AF, denoted as $\mathcal{E}_S(\mathrm{AF})$. According to this assumption, in order to define a general notion of skepticism between argumentation semantics, we rely on a more basic relation R between extensions. Given two semantics S_1 and S_2, S_1 is *more skeptical* than S_2, denoted as $S_1 \trianglelefteq S_2$, iff, for any argumentation framework AF, $\mathcal{E}_{S_1}(\mathrm{AF}) R \mathcal{E}_{S_2}(\mathrm{AF})$, where R is a relation between sets of extensions called *basic skepticism relation*. The point is now identifying a suitable basic skepticism relation that can encode the intuitive notion of skepticism.

As a starting point, it seems natural to assume as a basic constraint that, for any possible R and for any argumentation framework AF, R entails

$$\bigcap_{E_1 \in \mathcal{E}_{S_1}(\mathrm{AF})} E_1 \subseteq \bigcap_{E_2 \in \mathcal{E}_{S_2}(\mathrm{AF})} E_2 \qquad (1)$$

Relation (1) corresponds to the fact that all arguments justified according to S_1 are also justified according to S_2: being less skeptical should in any case leave unaltered the commitment made about justified arguments (i.e., the arguments included in the intersection of all extensions whose justification state is therefore $JS1$). This may be considered as a sort of intuitive bound for any relation of skepticism. However, it should be noticed that this relation alone is not appropriate as a skepticism relation, since it can be seen that it may hold even between two semantics that are not comparable. In fact, condition (1) does not prevent that there is an argument α attacked by all the extensions of S_1 (i.e. whose justification state is $JS2$ according to S_1) which belongs instead to all the extensions of S_2 and is, therefore, justified (i.e. its state is $JS1$). For instance, this may happen since condition (1) is compatible with the following ones:

- for any extension of S_1 there is an argument β attacking α such that β is not included in $\bigcap_{E_1 \in \mathcal{E}_{S_1}(\mathrm{AF})} E_1$
- any such β is out from all extensions of S_2
- α is included in any extension of S_2

In addition to the above constraint, one might also expect that a suitable basic skepticism relation should induce a notion of skepticism between semantics that agrees, for the specific case of the grounded and preferred semantics, with the well-known relation discussed in the Section 2.

Using as a basis the fact that, for any argumentation framework, the grounded extension is included in all preferred extensions, one may consider a generalization to the case of two multiple-status semantics prescribing that the extensions of S_1 satisfy some constraint of inclusion in the extensions of S_2. A natural way of obtaining this generalization is given by the following basic skepticism relation R_1:

Definition 3 *Given two sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$, $\mathcal{E}_1 R_1 \mathcal{E}_2$ iff*

$$\forall E_2 \in \mathcal{E}_2 \; \exists E_1 \in \mathcal{E}_1 : E_1 \subseteq E_2$$

The corresponding relation between semantics is denoted as $\trianglelefteq_1$.

Relation $\trianglelefteq_1$ is in a sense unidirectional, since it only constrains the extensions of S_2, while $\mathcal{E}_{\mathrm{AF}}(S_1)$ may contain additional extensions unrelated to those of S_2. One may wonder whether a more symmetric relationship is more appropriate, where it is also required that any extension of S_1 is included in one extension of S_2. To this purpose, we introduce the following definition:

Definition 4 *Given two sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$, $\mathcal{E}_1 R_2 \mathcal{E}_2$ iff*

$$\forall E_2 \in \mathcal{E}_2\ \exists E_1 \in \mathcal{E}_1 : E_1 \subseteq E_2, \text{ and}$$
$$\forall E_1 \in \mathcal{E}_1\ \exists E_2 \in \mathcal{E}_2 : E_1 \subseteq E_2$$

The corresponding relation between semantics is denoted as $\trianglelefteq_2$.

From the definitions, it immediately follows that $\trianglelefteq_1$ entails (1), and of course this holds also for $\trianglelefteq_2$ since $\trianglelefteq_2$ entails $\trianglelefteq_1$. Moreover, note that if $\mathcal{E}_1 = \{E_1\}$, i.e. the first semantics S_1 is a single-status approach, both $\trianglelefteq_1$ and $\trianglelefteq_2$ are equivalent to $\forall E_2 \in \mathcal{E}_2\ E_1 \subseteq E_2$. Therefore, in particular, if S_1 and S_2 are the grounded and the preferred semantics respectively, then the traditional relation holding between grounded and preferred semantics is recovered. Therefore both relations are good candidates for further investigation.

In the following, we will refer to $\trianglelefteq_1$ as *weak skepticism relation* and to $\trianglelefteq_2$ as *strong skepticism relation*: their properties will be analyzed in the following sections.

5 CHARACTERIZING THE SKEPTICISM RELATIONSHIPS

As the properties of the relationships between semantics $\trianglelefteq_1$ and $\trianglelefteq_2$ directly derive from those of the underlying relationships between sets of extensions, we carry out our analysis on R_1 and R_2. First of all, let us check whether the relationships give rise to a partial order, i.e. whether they are reflexive, antisymmetric and transitive.

Proposition 1 *Both R_1 and R_2 are preorders, i.e. they are reflexive and transitive.*

Proof: Given a generic set of extensions $\mathcal{E}$, it is easy to see that $\mathcal{E} R_2 \mathcal{E}$, since on the basis of the definition of R_2 this condition is equivalent to $\forall E_2 \in \mathcal{E}\ \exists E_1 \in \mathcal{E} : E_1 \subseteq E_2$, and $\forall E_1 \in \mathcal{E}\ \exists E_2 \in \mathcal{E} : E_1 \subseteq E_2$, which is obviously true. In other words, R_2 is reflexive, and since R_2 entails R_1 the latter is reflexive as well.
As for transitivity of R_1, let us suppose that $\mathcal{E}_1 R_1 \mathcal{E}_2$ and $\mathcal{E}_2 R_1 \mathcal{E}_3$. Since $\mathcal{E}_2 R_1 \mathcal{E}_3$, we have that $\forall E_3 \in \mathcal{E}_3\ \exists E_2 \in \mathcal{E}_2 : E_2 \subseteq E_3$, which taking into account that $\mathcal{E}_1 R_1 \mathcal{E}_2$ entails that $\exists E_1 \in \mathcal{E}_1$ such that $E_1 \subseteq E_2 \subseteq E_3$. Therefore, $\mathcal{E}_1 R_1 \mathcal{E}_3$.
As for transitivity of R_2, taking into account its definition and the fact that R_1 is transitive, the conclusion follows from the fact that, if $\forall E_1 \in \mathcal{E}_1\ \exists E_2 \in \mathcal{E}_2 : E_1 \subseteq E_2$ and $\forall E_2 \in \mathcal{E}_2\ \exists E_3 \in \mathcal{E}_3 : E_2 \subseteq E_3$, then $\forall E_1 \in \mathcal{E}_1\ \exists E_3 \in \mathcal{E}_3 : E_1 \subseteq E_3$. □

A simple example reveals that R_2 is not antisymmetric. In fact, let us consider three extensions E_1, E_2 and E_3 such that $E_1 \subseteq E_2 \subseteq E_3$, and let $\mathcal{E}_1$ be $\{E_1, E_2, E_3\}$, $\mathcal{E}_2$ be $\{E_1, E_3\}$. It can be seen that $\mathcal{E}_1 R_2 \mathcal{E}_2$ and $\mathcal{E}_2 R_2 \mathcal{E}_1$, however $\mathcal{E}_1 \neq \mathcal{E}_2$. Since R_2 entails R_1, neither the latter is antisymmetric. Therefore, neither R_1 nor R_2 is a partial order. It has however to be noted that a partial order is obtained if an additional constraint concerning maximality of extensions is introduced.

Definition 5 *A set of extensions $\mathcal{E}$ is I-maximal iff $\forall E_1, E_2 \in \mathcal{E}$, if $E_1 \subseteq E_2$ then $E_1 = E_2$.*

Since the sets of extensions prescribed by semantics in the literature are typically I-maximal, this constraint appears very reasonable, then both R_1 and R_2 turn out to be partial orders:

Proposition 2 *Given two I-maximal sets of extensions $\mathcal{E}_1$ and $\mathcal{E}_2$, if $\mathcal{E}_1 R_1 \mathcal{E}_2$ and $\mathcal{E}_2 R_1 \mathcal{E}_1$ then $\mathcal{E}_1 = \mathcal{E}_2$.*

Proof: On the basis of the hypothesis, we have that

$$\forall E_2 \in \mathcal{E}_2\ \exists E_1 \in \mathcal{E}_1 : E_1 \subseteq E_2 \quad (2)$$
$$\forall E_1 \in \mathcal{E}_1\ \exists E_2 \in \mathcal{E}_2 : E_2 \subseteq E_1 \quad (3)$$

First, we prove that $\mathcal{E}_2 \subseteq \mathcal{E}_1$. On the basis of (2) and (3), $\forall E_2 \in \mathcal{E}_2\ \exists E_1 \in \mathcal{E}_1, E_2' \in \mathcal{E}_2 : E_2' \subseteq E_1 \subseteq E_2$. Now, the maximality property of $\mathcal{E}_2$ entails that $E_2' = E_2$, thus $E_1 = E_2$. As a consequence, $\forall E_2 \in \mathcal{E}_2\ E_2 \in \mathcal{E}_1$, i.e. $\mathcal{E}_2 \subseteq \mathcal{E}_1$.
Finally, reasoning in a symmetric way we also get $\mathcal{E}_1 \subseteq \mathcal{E}_2$, and therefore $\mathcal{E}_1 = \mathcal{E}_2$. □

However, the assumption that sets of extensions are I-maximal might be questioned. In this case, we can consider the equivalence classes formed by elements x, y such that xRy and yRx, which, as well known, are arranged in a partial order induced by the preorder R. As it will be shown in the following section, in case of the strong skepticism relation an interesting property holds: all the semantics belonging to the same equivalence class assign to arguments the same justification states. In the case of the weak relation, this property holds in a looser form, i.e. adopting a coarser classification of states.

6 THE STRONG RELATION

As stated in Section 2, a relation of skepticism between semantics is related to a classification of justification states with respect to different levels of commitment. The basic idea is that, if S_1 is more skeptical than S_2, then for any argument α its justification state according to S_2 is comparable to and not less committed than its justification state according to S_1. We now show that, in the case of the strong skepticism relation, such classification corresponds to the partial order (actually a meet semi-lattice) whose Hasse diagram is shown in Figure 4. This order will be denoted as $\leq_S$ in the following. Basically, arcs connect pairs of comparable states, and lower states are less committed than higher ones. For instance, if α is in $JS5$ according to S_1 then its justification state according to S_2 is $JS1$, $JS7$, $JS6$ or $JS5$ itself. Therefore, the minimally committed state is $JS3$, and all justification states are comparable to it, while $JS1$, $JS6$ and $JS2$ are maximally committed.

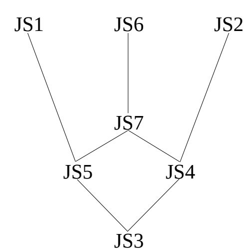

Figure 4. The $\leq_S$ semi-lattice of justification states.

Proposition 3 *Let us consider two semantics S_1 and S_2 such that $S_1 \trianglelefteq_2 S_2$. Then, for any argumentation framework $\mathrm{AF} = \langle \mathcal{A}, \rightarrow \rangle$ and for any argument $\alpha \in \mathcal{A}$, we have that $JS_u \leq_S JS_v$, where JS_u and JS_v are the justification states of α according to S_2 and S_1, respectively.*

Proof: According to the definition of strong skepticism relation, we have that:

$$\forall E_2 \in \mathcal{E}_{\rm AF}(S_2)\ \exists E_1 \in \mathcal{E}_{\rm AF}(S_1) : E_1 \subseteq E_2 \quad (4)$$

$$\forall E_1 \in \mathcal{E}_{\rm AF}(S_1)\ \exists E_2 \in \mathcal{E}_{\rm AF}(S_2) : E_1 \subseteq E_2 \quad (5)$$

On the basis of (4), if α is in $JS1$ ($JS2$) according to S_1, i.e. $\forall E_1 \in \mathcal{E}_{\rm AF}(S_1)$ α is in E_1 (definitely out from E_1), then we have also that $\forall E_2 \in \mathcal{E}_{\rm AF}(S_2)$ α is in E_2 (definitely out from E_2), i.e. α is in $JS1$ ($JS2$) according to S_2 as well. Therefore, $JS1$ and $JS2$ are maximal with respect to $\leq_S$.
On the basis of (5), if $\exists E_1 \in \mathcal{E}_{\rm AF}(S_1)$ such that α is in E_1, then it is also the case that $\exists E_2 \in \mathcal{E}_{\rm AF}(S_2)$ such that α is in E_2. Similarly, the existence of extensions from which α is definitely out is preserved. Exploiting these considerations, it is easy to see for instance that if α is in $JS5$ according to S_1 then, according to S_2, it must be in a state which preserves the existence of extensions where α is in. As a consequence, $JS5 \leq_S JS1$, $JS5 \leq_S JS7$, $JS5 \leq_S JS6$, and $JS5 \leq_S JS5$. In an analogous way, the following constraints can be derived: $JS4 \leq_S JS2$, $JS4 \leq_S JS7$, $JS4 \leq_S JS6$, and $JS4 \leq_S JS4$, $JS7 \leq_S JS6$, and $JS7 \leq_S JS7$.
As to $JS6$, the above considerations entail that, if the justification state of α prescribed by S_1 is $JS6$, then the justification state prescribed by S_2 is either $JS6$ or $JS7$. However, the latter is excluded by (4), which entails that $\forall E_2 \in \mathcal{E}_{\rm AF}(S_2)$ α is either in E_2 or definitely out from E_2. Therefore, $JS6$ is maximal with respect to $\leq_S$. □

Corollary 1 *Given two semantics S_1 and S_2 such that $S_1 \trianglelefteq_2 S_2$ and $S_2 \trianglelefteq_2 S_1$, for any argumentation framework $\rm AF = \langle \mathcal{A}, \rightarrow \rangle$ they assign the same justification state to any argument of $\mathcal{A}$.*

Proof: The claim easily follows from the constraints of the semi-lattice $\leq_S$. For instance, if α is in $JS5$ according to S_1, then it must be in $JS1$, $JS5$, $JS6$, or $JS7$ according to S_2. However, only $JS5$ is possible in order to satisfy the mutual constraints. This reasoning can be applied to all the states. □

An intuitive interpretation of the semi-lattice $\leq_S$ is based on the fact that, at the level of a single extension, the states *in* and *definitely out* are definitely committed, while the state *provisionally out* is not. Therefore, at the global level, the ordering is based on the presence of extensions from which an argument is provisionally out (e.g. the minimal state is $JS3$, i.e. provisionally out from all the extensions). This gives rise to the rather articulated set of constraints represented by $\leq_S$, which in turn corresponds to impose relatively strong requirements to ensure that two semantics are comparable. Consider in particular the fact that $JS6$ is maximal: as a consequence a semantics where all extensions are in agreement about a given argument α (e.g. α is in $JS1$) is not comparable with a semantics where at least two extensions do not agree but are all definitely committed about α. To make this observation concrete, consider the case of a credulous semantics that selects arbitrarily a preferred extension as its unique extension. While intuitively one might argue that such a credulous semantics is less skeptical than preferred semantics, it is easy to see that these two semantics are not comparable according to $\leq_S$. As an example, consider the classical Nixon Diamond, namely the argumentation framework $\mathrm{AF}_{ND} = \langle \{\alpha, \beta\}, \{\alpha \rightarrow \beta, \beta \rightarrow \alpha\} \rangle$, and suppose that the credulous semantics selects $\{\alpha\}$ as its unique extension. Both α and β are in $JS6$ according to preferred semantics, while α is in $JS1$ and β in $JS2$ according to credulous semantics. As $JS6$ is not comparable with $JS1$ or $JS2$ according to $\leq_S$, also the two semantics are not comparable.

The less restrictive relation $\trianglelefteq_1$ supports an alternative perspective in this respect.

7 THE WEAK RELATION

The weak skepticism relation $\trianglelefteq_1$ gives rise to the simpler partial order whose Hasse diagram is shown in Figure 5, and which will be denoted as $\leq_W$ in the following. In particular, JS_{3457} denotes the disjunction of the states listed in the subscript.

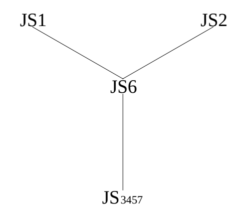

Figure 5. The $\leq_W$ semi-lattice of justification states.

Proposition 4 *Let us consider two semantics S_1 and S_2 such that $S_1 \trianglelefteq_1 S_2$. Then, for any argumentation framework $\rm AF = \langle \mathcal{A}, \rightarrow \rangle$ and for any argument $\alpha \in \mathcal{A}$, we have that $JS_u \leq_W JS_v$, where JS_u and JS_v are the justification states of α according to S_2 and S_1, respectively.*

Proof: On the basis of the definition of weak skepticism relation, it is the case that

$$\forall E_2 \in \mathcal{E}_{\rm AF}(S_2)\ \exists E_1 \in \mathcal{E}_{\rm AF}(S_1) : E_1 \subseteq E_2 \quad (6)$$

This entail that if α is in (definitely out from) all the extensions of S_1 the same situation holds with reference to the extensions of S_2. Therefore, if α is in $JS1$ ($JS2$) according to S_1 then it is in $JS1$ ($JS2$) also according to S_2. Moreover, if α is in $JS6$ then in particular for any extension $E_1 \in \mathcal{E}_{\rm AF}(S_1)$ it is either in E_1 or definitely out from E_1. On the basis of (6), this also holds for any extension $E_2 \in \mathcal{E}_{\rm AF}(S_2)$, entailing that the justification status according to S_2 is $JS1$, $JS2$ or $JS6$. No other constraints can be derived, giving rise to the minimal aggregated state JS_{3457}. □

It is evident that $\trianglelefteq_1$ gives rise to a coarser classification of states, as all the states such that there is an extension from which an argument is provisionally out (i.e. $JS3$, $JS4$, $JS5$ and $JS7$) are no more distinguished and collapse in JS_{3457}. On the other hand, $JS6$ has a different role from that one it has in $\trianglelefteq_2$ since it is now regarded as less committed than $JS1$ and $JS2$. As a consequence, the credulous semantics described above turns out to be less skeptical than preferred semantics according to $\trianglelefteq_1$.

Corollary 2 *Given two semantics S_1 and S_2 such that $S_1 \trianglelefteq_1 S_2$ and $S_2 \trianglelefteq_1 S_1$, for any argumentation framework $\rm AF = \langle \mathcal{A}, \rightarrow \rangle$ they assign the same justification state (in the coarser classification) to any argument of $\mathcal{A}$.*

Proof: As in Corollary 1. □

8 CONCLUSIONS

In this paper we have presented a first step towards the definition of a general framework for the analysis of the skepticism relation between argumentation semantics. This issue has been mainly considered in previous literature with reference to the debate concerning unique vs. multiple-status approaches. In the context of inheritance systems, skeptical reasoning has been identified with the production of a unique extension, while credulous reasoning with the production of multiple extensions [15, 8], however the use of these terms has not been uniform in the literature. For instance, in [12] the term "credulous" is used to qualify the behavior of a reasoner who chooses one of the multiple existing extensions arbitrarily, to solve impasses caused by ambiguous situations: in this respect, Pollock claims that "credulous reasoners are just wrong", since such an approach stems from a confusion of epistemic reasoning with practical reasoning. On the other hand, in [10, 14] the term "directly skeptical" is associated with single status approaches since they compute the justification states of arguments without resorting to multiple extensions. It is then proved that a directly skeptical approach is inherently unable to capture as justified all the arguments and conclusions that are justified in a multiple-status approach. While this has been commonly considered as a demonstration that only multiple-status approaches can properly deal with some particular reasoning cases concerning floating arguments, it has been recently claimed in [7] that the treatment of so-called floating conclusions needs to be more skeptical in some examples. In the extensive survey of argumentation by Prakken and Vreeswijk [13] the distinction between skeptical and credulous reasoning is related only to the justification states prescribed by a semantics, independently of the adoption of a unique or multiple status approach. The analysis carried out in this paper follows this perspective, by introducing an articulated classification of justification states and providing two relations of skepticism referring to generic sets of extensions. Their definitions therefore do not rely on any feature of specific proposals nor on the adopted approach and are applicable to any semantics fitting within Dung's framework.

The study of a generic cautiousness relationship, with similar goals of generality, has also been undertaken in [11] with the aim of partially ordering, with respect to skepticism, consequence relations between premises and conclusions. Our work focuses instead on an ordering among argumentation semantics rather than among inference mechanisms.

The importance of this topic is increased since the class of semantics of actual interest has been recently expanded by novel results in the field of argumentation [9, 1]. In particular, a recursive schema has been identified, which supports the definition of a space of argumentation semantics, including Dung's grounded and preferred semantics [2, 3] as well as a variety of novel proposals whose investigation has been recently undertaken [4]. Skepticism analysis may play a fundamental role in the analysis of this space.

Work in progress on the topic of skepticism involves several challenging issues that deserve further efforts. A key point is the connection between skepticism relation and relations between justifications states: are the latter a consequence of the former, as implicitly assumed in our present approach, or vice versa are the relations between justification states an abstract specification of (a class of) skepticism relations? Moreover, relation (1), introduced in Section 4 as a sort of intuitive bound for any possible skepticism relation, might be made more constrained; for example, one might impose a constraint not only on the set of definitely justified arguments, but also on the sets of definitely rejected arguments. Making this constraint more strict might support a more transparent definition of the concept of comparability between semantics.

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