An analysis of critical-link semantics with variable degrees of justification

Bin Wei[1] and **Henry Prakken** [2]

Abstract. The main aim of this paper is to critically examine Pollock's critical-link semantics with variable degrees of justification. We point out some possibly counterintuitive consequences of Pollock's definition of degrees of justification and propose a modified definition which avoids these consequences. We then modify the *ASPIC*$^+$ framework to allow for variable degrees of justification and then apply our modified way to compute these degrees.

1 Introduction

In most current AI approaches on modeling Argumentation, the justification status of arguments and conclusions is an all-or-nothing affair, but in many realistic applications, such as legal reasoning about evidence or other applications of epistemic reasoning, it is natural to regard them as justified to variable degrees. Pollock moddelled this in his so-called critical-link semantics in [1] and [2].

Pollock introduced variable justification degrees to account for the so-called "diminishing" effect of attempted defeaters that are weaker than their target. In such cases Pollock wanted to model that the attempted defeaters can still weaken the degree of justification of their target. The present paper aims to contribute to such a study by critically examining Pollock's proposal. In particular, we will argue that Pollock's approach in some cases gives counterintuitive outcomes, then modify his account in a way that avoids these outcomes. At the end, we will briefly discuss how Pollock's ideas and our modifications can be incorporated in the *ASPIC*$^+$ framework for structured argumentation recently proposed by [3].

This paper is organized as follows. In Section 2, we first summarize Pollock's semantics. In Section 3, we then discuss some arguably counterintuitive outcomes, present our revised definitions and show that they avoid these outcomes. In section 4, we discuss how to transfer the revised semantic into *ASPIC*$^+$ framework. Finally, we conclude in Section 5.

2 Semantics

In this section we present Pollock's critical-link semantics with variable degrees of justification, preceded by a brief overview of his [4] multiple-assignment semantics.

2.1 Basic features

In Pollock's account of defeasible reasoning, reasoning proceeds from a knowledge base of classical-logic formulas by chaining reasons into inference graphs, where all reasons are either deductive or defeasible. Only applications of defeasible reasons can be defeated, and there are two kinds of defeaters: *rebutting* defeaters attack the conclusion of a defeasible inference by favoring a conflicting conclusion, while *undercutting* defeaters attack the defeasible inference itself, without favouring a conflicting conclusion.

More precisely, Pollock assumes as given a knowledge base of first-order formulas and two sets of deductive and defeasible reasons, which technically are inference rules. Pollock then considers arguments, which are sequences of *argument lines*. The strength of an element φ of the knowlege base is below written as $\delta(\varphi)$ while the strength of a reason r will be written as $\rho(r)$.

Definition 2.1. *An argument line is a tuple (φ, r, L, s), where φ is a proposition, r is the reason applied to infer φ, L is the set of preceding lines from which φ is inferred, and s is the line's strength*[3].

Definition 2.2. *An argument line (φ, r, L, s) defeats an argument line (φ', r', L', s') iff r' is a defeasible rule, and $s \geq s'$, and $\varphi = \neg\varphi'$ or $\varphi = \neg r'$ (here $\neg r$ is shorthand for saying that the antecedents of rule r do not support its consequent).*

Definition 2.3. *For any argument line $l = (\varphi, r, L, s)$ (where $L = \{l_1, \ldots, l_n\}$) the strength $s(l)$ is inductively defined as follows:*

- *If l takes φ from the knowledge base, then $s(l) = \delta(\varphi)$.*
- *Otherwise, $s(l) = \min\{\rho(r), s(l_1), \ldots, s(l_n)\}$.*

With respect to accrual of arguments for the same conclusion, Pollock proposed that if we have two separate undefeated arguments for a conclusion, the degree of justification for the conclusion is the maximum of the strengths of the two arguments.

2.2 Multiple assignment semantic

In [4] Pollock considers inference graphs, where the nodes represent the propositions inferred from which they are inferred, support-links tie nodes to the nodes, and defeat-links indicate defeat relations between nodes. These links relate their roots to their targets. The root of a defeat-link is a singe node, while the root of a support-link is a set of nodes. He then proposes a labeling approach to define the justification status of nodes and propositions.

Definition 2.4. *A node of the inference-graph is* initial *iff its node-basis and list of node-defeaters is empty, where*

- *The* node-basis *of a node is the set of roots of its support links.*

[1] The Institute of Logic and Cognition, Sun Yat-Sen University, Guangzhou, China. email: srsysj@gmail.com

[2] Department of Information and Computing Sciences, Utrecht University and Faculty of Law, University of Groningen, The Netherlands

[3] Below the strength of argument line l will sometimes be written as $s(l)$.

- *The* node-defeaters *are the roots of the defeat links having the node as their target.*

Definition 2.5. *An assignment σ of defeated and undefeated to a subset of the nodes of an inference-graph is a* partial status assignment *iff:*

- *σ assigns undefeated to any initial node;*
- *σ assigns undefeated to a non-initial node α iff σ assigns undefeated to all the members of the node-basis of α and σ assigns defeated to all node-defeaters of α ;*
- *σ assigns defeated to a non-initial node α iff either σ assigns defeated to a member of the node-basis of α or σ assigns undefeated to a node-defeater of α.*

Definition 2.6. *Assignment σ is a* status assignment *iff σ is a partial status assignment and σ is not properly contained in any other partial status assignment.*

Definition 2.7. *A node α of an inference graph is undefeated iff every status assignment to the inference graph assigns undefeated to α; otherwise α is defeated.*

2.3 Critical-link semantics with variable degrees of justification

The core idea of critical-link semantics [1, 2] is to build new inference-graphs as subgraphs of the original inference graph and assign various statuses to initial nodes in different cases. This idea is formally defined as follows:

Definition 2.8. *An inference/defeat-path from a node φ to a node θ is a sequence of support-links and defeat-links such that (1) φ is a root of the first link in the path; (2) θ is the target of the last link in the path; (3) the root of each link after the first member of the path is the target of the preceding link; (4) the path does not contain an internal loop, i.e., no two links in the path have the same target.*

Definition 2.9. *A node θ of an inference graph is φ-dependent iff there is an inference/defeat-path from φ to θ.*

Definition 2.10. *A circular inference/defeat-path from a node φ to itself is an inference/defeat-path from φ to φ via a defeater of φ.*

Definition 2.11. *A defeat-link is φ-critical iff it is a member of some minimal set of defeat-links such that removing all the defeat-links in the set suffices to cut all the circular inference/defeat-paths from φ to φ.*

Definition 2.12. *If φ is a node of an inference graph G, then G_φ is the inference graph that results from (1) deleting all φ-critical defeat-links from G and (2) making all members of the node-basis of φ initial nodes in G_φ and (3) making all φ-independent nodes initial-nodes in G_φ with stipulated defeat-statuses the same as their defeat-statuses in G.*

We next discuss how Pollock uses his critical-link semantics to define variable degrees of justification. A main motivation of the idea that propositions should have variable degrees of justification is Pollock' notion of a *diminisher*. A diminisher is a defeater of a node that is weaker than its target, which is able to diminish the *degree* of justification of its target.

For the sake of the mathematics of diminishers, Pollock proposed that there exists a function $\diamond$[4] such that given two argument lines that rebut one another, if their strengths are x and y, the degree of justification for the conclusion of the former is $x \diamond y$, while the degree of justification for conclusion of y is $y \diamond x$. He assumed that "the degree of justification can be measured using real numbers, possibly augmented with ∞, i.e., 'the extended real numbers'. More precisely, the degrees of justification fall in some interval $[o, \theta]$, where $0 \leq o \leq \theta \leq \infty$. o corresponds to no justification, and θ to perfect justification, presumably only possible for necessary truths.". Then Pollock defined mathematical properties of $\diamond$ as follows:

Definition 2.13. *[Mathematics of $\diamond$]*
(A1) $\diamond$ is continuous on the interval $[o, \theta]$.
(A2) If $\theta > \alpha > \beta > o$, then $\alpha > \alpha \diamond \beta > o$.
(A3) If $\theta > \alpha > \beta > \gamma > o$, then $\alpha \diamond \beta < \alpha \diamond \gamma$ and $\alpha \diamond \gamma < \beta \diamond \gamma$.
(A4) If $\theta \geq \alpha \geq \beta > o$, then $\beta \diamond \alpha = o$.
(A5) If $\theta \geq \alpha > o$, then $\alpha \diamond o = \alpha$.
(A6) If $\theta > \alpha$ and β and γ are in $[o, \theta]$, then $(\alpha \diamond \beta) \diamond \gamma = (\alpha \diamond \gamma) \diamond \beta$.

Pollock proved that if (A1) $-$ (A6) hold, then $\diamond$ has a very simple representation as follows:

Definition 2.14. *[Representation of $\sim$]*

$$x \sim y = \begin{cases} x - y & \text{if } y < x < \infty \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Definition 2.15. *[Computation of degree of justification]*
(DJ) If P is inferred from the basis $\{B_1, \ldots, B_n\}$ in an inference-graph G in accordance with a reason of strength ρ, $D_1, \ldots, D_k$ are the P-independent defeaters for P , and $D_{k+1}, \ldots, D_m$ are the P-dependent defeaters of P, then $J(P, G) = \min\{\rho, J(B_1, G), \ldots, J(B_n, G)\} \sim \big[\max\{J(D_1, G), \ldots, J(D_k, G)\} + \max\{J(D_{k+1}, G_P), \ldots, J(D_m, G_P)\}\big]$.

DJ is a computation for "collaborative defeat", where the nodes are defeated by both node-dependent and node-independent defeaters.

3 Problem cases and modifications

In this section, we discuss some possible problems of Pollock's critical-link semantics with variable degrees of justification, by analyzing some problem cases.

3.1 Problem case on diminishers

The first problem concerns some arguably counter-intuitive consequences of the mathematical properties and representation of the function $\diamond$. We present an example and discuss why the outcomes may be counter-intuitive, and then modify some properties of $\diamond$ and choose another definition for $\sim$ to represent $\diamond$.

Consider rebutting defeaters in Figure 1. Let P be "Jones says that it is not raining", R be "Smith says that it is raining", and Q be "It is raining". Let us first assume that Smith and Jones as equally reliable. Then according to Pollock both Q and $\neg Q$ should be defeated. If we apply Definition 2.15 and again assume that the degrees of justification of the initial nodes are at least as great as the strengths of reasons, then we have $J(Q, G) = J(\neg Q, G) = 0$. Assume next that Smith is much more reliable than Jones: then Q defeats $\neg Q$ while $\neg Q$ diminishes Q: by Definition 2.15 we have $J(\neg Q, G) = 0$ and $J(Q, G) = J(R, G) \sim J(P, G) > 0$.

The arguably counter-intuitive consequence is that node $\neg Q$ has in both cases the same degree of justification, namely, 0, while yet in the

[4] Pollock added the mathematical analysis in his extended version, http://oscarhome.soc-sci.arizona.edu/ftp/PAPERS/Degrees.pdf.

second case the degree of justification of Q is higher than in the first case. Thus intuitively, although node $\neg Q$ is in the first case not accepted, it is still much more reliable than in the second case. Thus the degrees of justification of nodes in cases of symmetric defeat should be greater than the ones in cases of asymmetric defeat. Moreover, the first case is similar to "zombie arguments"[5]: although the arguments are defeated, they can still affect another arguments. In other words, the node $\neg Q$ in the first case still has ability to attack or support other nodes, but the node $\neg Q$ in the second case does not. So it is necessary to make a difference between the degrees of justification of nodes in these two cases.

3.2 Problem case on "presumptive defeat"

The previous point can be further developed in a discussion of ambiguity blocking vs. ambiguity propagating (by Pollock called "presumptive defeat"in [1]). Consider again Figure 1 but let now Q stand for "Rain was predicted by the morning weather forecast", P for "Jones says that no rain was predicted by the morning weather forecast", R for "Smith says that rain was predicted by the morning weather forecast", S for "It will rain"and A for "rain was predicted by the afternoon weather forecast". Suppose again that the reason strengths are at least as great as those of the initial nodes and suppose that P and R are equally strong. Then according to Pollock's new approach the degree of justification of all of Q, $\neg Q$ and $\neg S$ equals 0, so that $\neg S$ cannot diminish or defeat S. However, according to Section 3.1 the degrees of justification of Q and $\neg Q$ should be greater than 0, and this has the consequence that $\neg S$ potentially has the force to diminish or even defeat S.

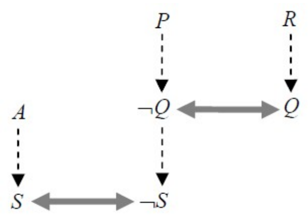

Figure 1. Presumptive defeat

3.3 Problem case on undercutters

Next we discuss a problem of the computation principle *DJ* by arguing that it gives an unnatural treatment of the effect of undercutters on the degree of justification of their target. Consider an inference graph with undercutter, let P be "Jones says that it is raining" and Q be "It is raining", R be "Smith says that John always lies" and $P \otimes Q$ be "John is lying" means "P does not guarantee Q". Note that node $P \otimes Q$ attacks the connection between node P and node Q, so the strength of node $P \otimes Q$ should arguably directly weaken the strength of the reason from P to Q and only indirectly weaken the strength of node Q. In other words, the strength of an undercutting node should be in comparison with the strength of the reason it undercuts rather than with the strength of the node it attacks. However, in Pollock's definitions this is not the case.

3.4 Modified definition of representation

In his final paper [6], Pollock reconsidered the problem of degrees of justification. He measured degrees of justification using numbers in the interval $[0, 1]$, for which reason we henceforth choose the scale as $[0, 1]$. From assumptions (A2) and (A4) it's clear to show that Pollock meant to design the function to capture the diminishers diminish nodes without completely defeating and diminishers diminish nodes with completely defeating. However, some assumptions of mathematical properties of operator are counter-intuitive and should be revised in order to avoid the above problems.

Firstly, according to the above analysis on diminisher and "presumptive defeat". Assumption $(A4)$ should be modified as follows:

(A4) If $\theta > \alpha > \beta > o$, then $\beta \diamond \alpha = o$.

(A4') If $\theta > \alpha = \beta \geq o$, then $\beta \diamond \alpha \geq o$.

These two revised assumptions that the degrees of justification of nodes in defeat cycles should be greater than 0.

Secondly, according to the above analysis of diminishers, the degree of justification of diminished node reduces to real number 0 when the strength of the diminishing node with completing defeating is approaching to the strength of the diminished node. However, the degree of justification of the diminished node would be definitely greater than 0 in accordance with (A4') if the strengths of the rebutting defeaters are equal. Therefore, the representation is not continuous on the whole interval $[0, 1]$, since any point (x_0, y_0) that satisfies $x_0 = y_0$ would be a discontinuous point. But Pollock wanted that diminishing nodes without completely defeating and diminishing nodes with completely defeating are, respectively, continuous. Therefore, we use $f(x, y)$ to present a diminishing node with degree y that completely defeats a diminished node with degree x, and use $g(x, y)$ to present a diminishing node with degree y that does not completely defeat a diminished node with degree x. We replace assumption (A1) by saying that $f(x, y)$ and $g(x, y)$ are continuous.

Thirdly, the degree of justification for a diminished node should be the strength of this node decremented by an amount determined by the strength of the diminishing node. Moreover, the strength of a node as conclusion is determined by the strength of its reason and the strength of its node as premise. Rebutting defeaters or undercutting defeaters can both act as diminishers but their influences on diminished nodes are different. Undercutting defeaters weaken the strength of the reason they attack, while rebutting defeaters directly weaken the strength of the node as conclusion. Therefore, the order in which undercutting defeaters and rebutting defeaters as diminishers are applied to an argument makes a difference to the degrees of justification, and this in turn means that A(6) is invalid.

In sum, our analysis in Sections 3.1-3.3 makes that assumption (A4) must be modified while assumptions (A1) and (A6) cannot hold. We now define a new representation $\sim$ for operator $\diamond$, which matches the above-revised assumptions. Let us define:

$$x \sim y = \begin{cases} x(1-y) & \text{if } y \leq x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

It's easy to prove that the new function satisfies the revisions of Pollock's conditions:

(A1) $f(x, y) = x(1 - y)$ and $g(x, y) = 0$ are continuous on the interval $[0, 1]$

(A2) If $1 > x > y > 0$, then $x > x \sim y > 0$.

(A3) If $1 > x > y > z > 0$, then $x \sim y < x \sim z$ and $x \sim z > y \sim z$

(A4) If $1 > x > y > o$, then $y \sim x = 0$.

(A4') If $1 > x = y \geq 0$ then $x \sim y \geq 0$

(A5) If $1 \geq x > 0$, then $x \sim 0 = x$

3.5 Modified definition of variable degrees of justification

The revised idea for the problem case of undercutters is that the degree of justification of node P equals the minimum of the strength of reason after being diminished and the degrees of justification of its premises. Then the computation for nodes not in a circular path can be modified as follows: If P has P-independent defeaters $D_1, \ldots, D_k$ in G and has no P-dependent defeaters, then $J(P,G) = \min\{(\rho \sim \max\{J(D_1,G), \ldots, J(D_k,G)\}), J(B_1,G), \ldots, J(B_n,G)\}$.

We next discuss the case where a node P is defeated by both P-dependent defeaters and P-independent defeaters. We propose that these two kinds of defeaters can unite to defeat node P with a double counting, but computing it with P-independent defeaters firstly and then continue to compute it with P-dependent defeaters. The final computation can be modified as follows:

Definition 3.1. *[Modified Computation] If P is inferred from the basis $\{B_1, \ldots, B_n\}$ in an inference-graph G in accordance with a reason of strength ρ, $D_1, \ldots, D_k$ are the P-independent defeaters for P, and $D_{k+1}, \ldots, D_m$ are the P-dependent defeaters of P, then $J(P,G) = \min\{(\rho \sim \max\{J(D_1,G), \ldots, J(D_k,G)\}), J(B_1,G), \ldots, J(B_n,G)\} \sim \max\{J(D_{k+1},G_P), \ldots, J(D_m,G_P)\}$*

For instance, in Figure 2, node $\neg S$ is S-dependent, node S is $\neg S$-dependent and node $Q \otimes S$ is S-independent. Let $J(P,G) = 0.15$, $J(Q,S) = J(R,G) = 0.8$ and the reasons are equally strong: $\rho = 0.9$. Then $J(Q \otimes S, G) = 0.8$, $\rho \sim J(Q \otimes S, G) = 0.18$, $J(\neg S, G_S) = 0.15$, $J(S,G) = \min\{\rho \sim J(Q \otimes S, G), J(Q,G)\} \sim J(\neg S, G_S) = 0.18 \sim 0.15 = 0.153$, and $J(S, G_S) = 0.18$, $J(\neg S, G) = \min\{\rho, J(P,G)\} \sim J(S, G_{\neg S}) = 0.15 \sim 0.18 = 0$.

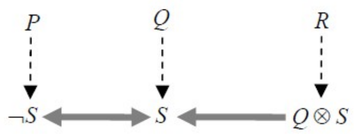

Figure 2. Inference graphs with collaborative defeaters

3.6 Solution to the problem cases

We now show that the new definition avoids the arguably counter-intuitive outcomes we described above. We do this by analyzing the example of presumptive defeat, which includes the problem case of diminishers. Consider again the example in Figure 1. In the multiple-assignment semantics in [4], $\neg Q$ has the ability to support $\neg S$ if $\neg Q$ is assigned undefeated in the partial status assignment and $\neg Q$ has no ability to support S if $\neg Q$ is assigned defeated in the other partial status assignment. With our new definition of $\sim$ the outcome is different. For simplicity, we again assume that the strengths of reasons are at least as great as the degrees of justification of the initial node. Then the computation of $J(\neg S, G)$ can be concluded as follows: $J(\neg S, G) = \min\{\rho, J(\neg Q, G)\} \sim J(S, G_{\neg S}) = J(\neg Q, G) \sim J(A,G) = (J(P,G) \sim J(R,G)) \sim J(A,G)$.

We discuss the possible degrees of justification of $\neg Q$ and $\neg S$. $\neg Q$ has ability to support $\neg S$ iff $J(P,G) \geq J(R,G)$. Hence, $J(\neg S,G) > 0$ iff $\neg Q$ has ability to support $\neg S$ and $J(P,G)(1 - J(R,G)) \geq J(A,G)$. Otherwise, $J(\neg S,G) = 0$. For instance, let $J(P,G) = J(R,G) = 0.8$, $J(A,G) = 0.1$ and the reason-strengths are equally strong: $\rho = 0.9$, then $J(\neg Q,G) = \min\{\rho, J(P,G)\} \sim J(Q, G_{\neg Q}) = J(P,G) \sim J(R,G) = 0.16$; $J(Q,G) = \min\{\rho, J(R,G)\} \sim J(\neg Q, G_Q) = J(R,G) \sim J(P,G) = 0.16$; $J(\neg S,G) = \min\{\rho, J(\neg Q,G)\} \sim J(S, G_{\neg S}) = J(\neg Q,G) \sim J(A,G) = 0.144$.

Apparently, $\neg Q$ has the power to support $\neg S$ and $\neg S$ therefore has the ability to defeat or support another nodes. Moreover, if we let $J(P,G) < J(R,G)$ or $J(P,G)(1 - J(R,G)) < J(A,G)$, the justification of $\neg S$ equals 0.

4 Variable degrees of justification in the *ASPIC*$^+$ framework

The idea of critical-link semantics with variable degrees of justification is a general theory and can be applied in other argumentation formalisms as well. We will discuss the computation of degrees of justification combined with *ASPIC*$^+$, using the new notion of an argument graph. We regard the degree of justification of an argument[5] as the variable degree for accepting or rejecting the argument from a cognitive perspective. We next give some new definitions that are useful in our modification associated with *ASPIC*$^+$.

Definition 4.1. *[Argument strength] $\mathcal{V}$ is a function to evaluate the strength of an argument with conditions as follows:*

- *if $A \in K$, then $\mathcal{V}(A) = \eta(A)$, where η is a function that assigns the degrees of acceptability of the premises in an argument, modeled as $\eta(A) : 2^{Prem(A)} \to [0,1]$.*
- *if A is the form $A_1, \ldots, A_n \to/\Rightarrow \varphi$, then $\mathcal{V}(A) = \min\{\mathcal{V}(A_1), \ldots, \mathcal{V}(A_n), \nu(\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \to/\Rightarrow \psi)\}$, where ν is a function assigns the degree of support from antecedent to consequent in a strict or defeasible inference, modeled as: $\nu(\delta) : \delta \to [0,1)$, where $\delta \in \mathcal{R}_s$ and $\nu(\delta) = 1$, where $\delta \in \mathcal{R}_d$.*

Definition 4.2. *[Maximal proper subargument] Argument A is a* maximal proper subargument *of B iff A is a subargument of B and there does not exist any proper subargument C of B such that A is a proper subarugment of C.*

Definition 4.3. *[Direct attacking] Argument A* directly attacks *argument B iff A rebuts or undercuts B on B; otherwise A indirectly attacks B.*

Definition 4.4. *An* argument graph *G is a labeled, finite, directed, bipartite graph, consisting of argument nodes and attacking links indicating attacking relationships between argument nodes and proper subargument links indicating connecting subargument relationships between an argument and its proper superarguments.*

The attacking links relate their roots to their targets and the root of an attacking link is an attacker in the graph, while the proper subargument links relate their roots to their targets and the root is the proper subargument of its target or the target is the proper superargument of its root in graph. In the diagrams of argument graphs, argument are

[5] We assume that the degree of justification of one argument equals the degree of justification of its conclusion.

displayed as dots, attacking links are indicated using ordinary arrowheads, while proper subargument links are indicated using closed-dot arrowheads. The initial arguments in G can be defined as follows:

Definition 4.5. *An argument is initial in G iff it is not the target of any attacking link or proper subargument link.*

Consider and Pollock's inference graph in Figure 1. We assume arguments in *ASPIC*$^+$ framework as $B : B_1 \Rightarrow \neg S$; $B_1 : B_2 \Rightarrow \neg Q$; $B_2 : P$; $C : C_1 \Rightarrow Q$; $C_1 : R$; $D : D_1 \Rightarrow S$; $D_1 : A$. We show the arguments in Figure 3. Note that C directly rebuts B_1 and indirectly rebuts B, B directly rebuts D. Moreover, nodes B_2, C_1 and D_1 are initial arguments.

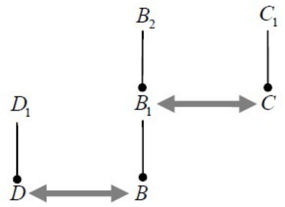

Figure 3. Argument graph

Definition 4.6. *An $argument\ path$ $P(A, B)$ from argument A to argument B in graph G is a sequence of attacking links and proper subargument links $\langle L_1, \dots, L_n\rangle$, such that*

1. *Argument A is the initial argument that there is no argument in graph G attacks A;*
2. *there exists arguments $B_1, \dots, B_{n-1}$, such that $L_1 = (A, B_1)$, $L_{i+1} = (B_i, B_{i+1})$, and $L_n = (B_{n-1}, B)$, where (A, B) means the attack link or proper subargument link from A to B.*

Next we will make our approach simpler than Pollock's by defining the notions of a basic set and its extension instead of the notions of node-dependent and node-critical links.

Definition 4.7. *The notions of basic set and critical extension can be defined as follows:*

1. *A set of attack links is a* basic set *of argument A in graph G iff removing all members of the set suffices to cut all cycles from A to A.*
2. *A set of attack links is a* critical extension *of argument A in graph G iff it is a minimum basic set of argument of A in graph G.*

Proposition 1. *For any argument A in a circular path, there exists at least one basic set of A.*

Proposition 2. *For any attack link L in a circular path P, there exists at least one critical extension containing L.*

Corollary 1. *If an attack link does not occur on any circular path, then it does not belong to any critical extension.*

Definition 4.8. *Given a graph G, the new graph G_A is the argument-graph that results from removing all members in all critical extensions in graph G and making all arguments $B_1, \dots, B_n$ which are not in a defeat cycle initial with $J(B_i, G_A) = J(B_i, G)$.*

Definition 4.9. *[Justification computation]*

1. *If A is initial in G, then $J(A, G) = \mathcal{V}(A, G)$.*
2. *If A is initial in G_A, and $B_1, \dots, B_n$ are direct rebutters of A or undermining attackers in cycles from A to A, then $J(A, G) = \mathcal{V}(A, G) \sim \max\{J(B_1, G_A), \dots, J(B_n, G_A)\}$.*
3. *If A is not initial in G, and $A_1, \dots, A_n$ are the maximal proper subarguments of argument A, and ρ is the strength of $Toprule(A)$, $B_1, \dots, B_i$ are direct undercuters of A and $B_{i+1}, \dots, B_m$ are direct rebutters of A or undermining attackers in cycles from A to A, then $J(A, G) = \min\big\{(\rho \sim \max\{J(B_1, G), \dots, J(B_i, G)\}), J(A_1, G), \dots, J(A_n, G)\big\} \sim \max\{J(B_{i+1}, G_A), \dots, J(B_m, G_A)\}$.*

We define $x \sim y = x(1 - y)$, if $y \leq x < 1$, otherwise, $x \sim y = 0$ and $\max\{\emptyset\} = 0$. The computation is for argument attacked both by direct undercutters and direct rebutters or underminers in cycles. It unites and double counts the computation for arguments only attacked by direct undercutters and the computation for arguments only attacked by direct rebutters or underminers in cycles.

Finally, we illustrate the new definition by computing the degree of justification of argument B in Figure 3. Let $J(B_2, G) = 0.8$, $J(C_1, G) = 0.8$, $J(D_1, G) = 0.1$ and the reasons are equally strong: $\rho = 0.9$. It is clear that C directly rebuts B_1, then from (DJ), it follows $J(B_1, G) = \min\{\rho, J(B_2, G)\} \sim J(C, G_{B_1}) = J(B_2, G) \sim J(C_1, G) = 0.16$; we also have B directly rebuts D, then from (DJ), it follows $J(B, G) = \min\{\rho, J(B_1, G)\} \sim J(C, G_B) = J(B_1, G) \sim J(D_1, G) = 0.144$; Similar, $J(B, G_D) = 0.16$; $J(D, G) = \min\{\rho, J(D_1, G)\} \sim J(B, G_D) = J(D_1, G) \sim J(B, G_D) = 0$.

5 Conclusion

In this paper we studied the modelling of variable degrees of justification in argumentation. We pointed out some arguably counter-intuitive consequences of Pollock's critical-link semantics with variable degrees of justification and then presented some modifications that avoid these outcomes. Moreover, to illustrate the generality of Pollock' approach and our modifications, we also discussed how they can be combined with the *ASPIC*$^+$ framework. In future work we aim to investigate the properties of our definitions and to study their application to realistic examples, including problems of legal reasoning with evidence.

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