Defining the structure of arguments with AI models of argumentation

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Abstract. The structure of arguments is an important issue in the field of informal logic and argumentation theory. In this paper we discuss how the 'standard approach' of Walton, Freeman and others can be analysed from a formal perspective. We use the $ASPIC^+$ framework for making the standard model of argument structure complete and for introducing a distinction between types of individual arguments and types of argument structures. We then show that Vorobej's extension of the standard model with a new type of hybrid arguments is not needed if our formal approach is adopted.

1 Introduction

The structure of arguments is an important issue in the field of informal logic and argumentation theory. The main issue is to define the different ways in which premises and conclusions can be combined to generate different structural argument types. The 'standard approach' was introduced by Stephen N. Thomas in [1] and was further developed by, among others, Walton [2] and Freeman [3]. Vorobej [4] extended their approach with an additional argument type called "hybrid arguments". This paper aims to show how formal AI models of argumentation can be used to further extend and clarify these informal models of the structure of arguments. In particular, we argue that they have some limitations, since their classifications are incomplete and since they do not distinguish between types of individual arguments and structures consisting of several arguments. Moreover, we argue that Vorobej's proposal can be clarified by making a distinction between deductive and defeasible arguments.

We aim to achieve our aims by applying the $ASPIC^+$ framework of [6]. We use it to make three specific contributions: (1) to make the standard classifications complete; to (2) indicate and explain why convergent and divergent arguments are not arguments but argument structures; and (3) to indicate and explain why Vorobej's class of hybrid arguments is not needed if an explicit distinction is made between deductive and defeasible arguments.

This paper is organized as follows. In section 2, we introduce the standard informal model of argument structure and Vorobej's [4] extension with so-called hybrid arguments. In Section 3, we present a simplified version of the $ASPIC^+$ framework. We then use this framework in section 4 to complete the standard model and to distinguish between types and structures of arguments. In section 5, we discuss Vorobej's notion of hybrid arguments and how it can be captured in $ASPIC^+$. Section 6 concludes the paper.

2 Standard approaches to argument structure

We first introduce the main approaches to argument structures, notably the approach by e.g. Walton [2] and Freeman [3], which we will call the 'standard' approach and Vorobej's [4] extension with so-called hybrid arguments.

2.1 Standard approach

The standard approach to the structure of arguments was introduced by Stephen N. Thomas in [1]. He divided the arguments into (1) *linked arguments*, which means that every premise is dependent on the others to support the conclusion, (2) *convergent arguments*, which means that premises support the conclusion individually, (3) *divergent arguments*, which means that one premise supports two or more conclusions, and (4) *serial arguments*, which means that one premise supports a conclusion which supports another conclusion.

Walton then further discussed the structure of arguments in [2]. We present the informal definitions of the concepts of structures of arguments according to his latest description in [5]. The corresponding diagrams are shown in Figure 1.

Definition 1. *The types of arguments are informally defined as follows:*

- (1) An argument is a single argument iff it has only one premise to give a reason to support the conclusion.
- (2) An argument is a convergent argument iff there is more than one premise and where each premise functions separately as a reason to support the conclusion.
- (3) An argument is a linked argument iff the premises function together to give a reason to support the conclusion.
- (4) An argument is a serial argument iff there is a sequence $\{A_1, \ldots, A_n\}$ such that one proposition A_i acts as the conclusion drawn from other proposition A_{i-1} as premise and it also functions as a premise from which a new proposition A_{i+1} as conclusion is drawn.
- (5) An argument is a divergent argument iff there are two or more propositions inferred as separate conclusions from the same premise.
- (6) An argument is a complex argument iff it combines at least two arguments of types (2),(3),(4) or (5).

Example 1. Walton gives the following examples of, respectively, a convergent, divergent and linked argument:

 (A) Tipping makes the party receiving the tip feel undignified; (B) Tipping leads to an underground, black-market economy; (C) Tipping is a bad practice.

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- (2) (A) Smoking has been proved to be very dangerous to health;
 (B) Commercial advertisements for cigarettes should be banned;
 (C) Warnings that smoking is dangerous should be printed on all cigarette packages.
- (3) (A) Birds fly; (B) Tweety is a bird; (C) Tweety flies.



Figure 1. Structures of argument

In Example 1(1), the three statements form a convergent argument, since statements (A) and (B) function separately as a reason to support the conclusion (C). By contrast, in Example 1(2) these three statements form a divergent argument, since statement (B) and (C) are inferred as separate conclusions from the same premise (A). Finally, Example 1(3) is a linked argument, since neither premise alone gives any reason to accept the conclusion.

2.2 Hybrid arguments

In [4], Mark Vorobej argued that the standard approach needs to be extended with a class of hybrid arguments. To discucss this class, we must first present Vorobej's basic definitions of types of arguments.

Definition 2. An argument A is:

- simple iff A has exactly one conclusion. Otherwise, A is complex.
- convergent iff A is simple and each premise in A is relevant to C, where relevance is treated as a primitive dyadic relation obtaining in each instance between a set of propositions and a single proposition.

Definition 3. A linked set and linked argument are defined as follows:

- A set of premises Δ forms a linked set iff
 - (1) Δ contains at least two members;
 - (2) Δ is relevant to C, and
 - (3) no proper subset of Δ is relevant to C.
- An argument A is linked iff A is simple and each premise in A is a member of some linked set.

Vorobej then motivates this new class of hybrid arguments with examples like the following one.

Example 2. Consider example (F) as follows:

• (F): (1) All the ducks that I' ve seen on the pond are yellow. (2) I've seen all the ducks on the pond. (3) All the ducks on the pond are yellow.

Vorobej observes that (2) in isolation is not relevant to (3), so this is not a convergent argument. Secondly, (1) is relevant to (3), so (1) is not a member of any linked set, so this is also not a linked argument. Vorobej regards (F) as a hybrid argument, since (1) is relevant to the conclusion (3) and (2) is not relevant to the conclusion (3) but (1) and (2) together provide an additional reason for (3), besides the reason provided by (1) alone.

Vorobej provides the following definition of hybrid arguments in terms of a relation of supplementation between premises.

Definition 4. *The relation of supplementation and hybrid argument are defined as follows:*

- A set of premises Σ supplements a set of premises △ iff
 (1) Σ is not relevant to C;
 - (2) Δ is relevant to C;

(3) $\Sigma \cup \Delta$ offers an additional reason *R* in support of *C*, which Δ alone does not provide, and

(4) Σ and Δ are the minimal sets yielding R which satisfy clauses (1),(2) and (3).

• An argument A is a hybrid iff A is simple and contains at least one supplemented (or supplementing) set.

In Example 2 premise (2) supplements premise (1). The argument is therefore a hybrid argument.

3 The *ASPIC*⁺ framework

The *ASPIC*⁺ framework of [6] models arguments as inference trees constructed by two types of inference rules, namely, strict and defeasible inference rules. The framework has in [6, 7, 8, 9] been shown to capture a number of other approaches to structured argumentation, such as assumption-based argumentation [10], forms of classical argumentation [11] and Carneades [12]. In this paper we use a simplified version of *ASPIC*⁺ framework, with negation instead of an arbitrary contrariness function over the language, with just one instead of four types of premises, and without preferences.

Definition 5. [Argumentation system] An argumentation system is a tuple $AS = (\mathcal{L}, \mathcal{R})$ where

- *L* is a logical language closed under negation (¬). Below we write
 ψ = −*φ* when either *ψ* = ¬*φ* or *φ* = ¬*ψ*.
- $\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ is a set of strict (\mathcal{R}_s) and defeasible (\mathcal{R}_d) inference rules such that $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$.

Definition 6. [*Knowledge bases*] A knowledge base in an argumentation system $(\mathcal{L}, \mathcal{R})$ is a set $\mathcal{K} \subseteq \mathcal{L}$.

Arguments can be constructed step-by-step by chaining inference rules into trees. In what follows, for a given argument the function *Prem* returns all its premises, *Conc* returns its conclusion *Sub* returns all its sub-arguments, while *TopRule* returns the last inference rule applied in the argument.

Definition 7. [Argument] An argument A on the basis of a knowledge base \mathcal{K} in an argumentation system $(\mathcal{L}, {}^{-}, \mathcal{R})$ is:

- 1. φ if $\varphi \in \mathcal{K}$ with: $Prem(A) = \{\varphi\}$; $Conc(A) = \varphi$; $Sub(A) = \{\varphi\}$; TopRule(A) = undefined.
- 2. $A_1, \ldots, A_n \rightarrow \Rightarrow \psi$ if A_1, \ldots, A_n are arguments such that there exists a strict/defeasible rule $Conc(A_1), \ldots, Conc(A_n) \rightarrow \Rightarrow \psi$ in $\mathcal{R}_s/\mathcal{R}_d$.
 - $Prem(A) = Prem(A_1) \cup \ldots \cup Prem(A_n); \ Conc(A) = \psi;$ $Sub(A) = Sub(A_1) \cup \ldots \cup Sub(A_n) \cup \{A\}; \ TopRule(A) =$ $Conc(A_1), \ldots, Conc(A_n) \to \psi.$

An argument is *strict* if all its inference rules are strict and *defea-sible* otherwise.

Definition 8. [Maximal proper subargument] Argument A is a maximal proper subargument of B iff A is a subargument of B and there does not exist any proper subargument C of B such that A is a proper subargument of C.

Example 3. Consider a knowledge base in an argumentation system with $R_s = \{p, q \rightarrow s; u, v \rightarrow w\}$; $R_d = \{p \Rightarrow t; s, r, t \Rightarrow v\}$; $K = \{p, q, r, u\}$.

An argument for w is displayed in Figure 2. Strict inferences are displayed with solid lines and defeasible inferences with dotted lines. Formally the argument and its subarguments are written as follows:

 $A_1:p$ $A_6: A_1, A_2 \to s$ $A_7: A_3, A_4, A_6 \Rightarrow v$ $A_2:q$ $A_8: A_5 \to n$ $A_3 : r$ $A_4:t$ $A_9: A_8 \Rightarrow u$ $A_{10}: A_7, A_9 \to w$ $A_5:m$ We have that $Prem(A_{10}) = \{p, q, r, t, m\}$ $Conc(A_{10}) = w$ $Sub(A_{10}) = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$ $MaxSub(A_{10}) = \{A_7, A_9\}$ $Toprule(A_{10}) = \{u, v \to w\}$



Figure 2. An Argument

In $ASPIC^+$ there are three syntactic forms of *attacks*: an *undercutter* attacks the inference rule, a *rebuttal* attacks the conclusion, and an *underminer* attacks a premise. Rebutting and undercutting attacks can only be targeted at (conclusions of) defeasible inference rules. So the argument in Figure 2 can only be rebutted on the (inferences of) the conclusions v and u. Attacks combined with preferences defined by an argument ordering yield three kinds of defeat. For the formal definitions of attack and defeat see [6].

4 Types and structures of argument

We now give a new classification of arguments in terms of the $ASPIC^+$ framework and then define so-called argument structures, which are collections of arguments with certain features. We first define two kinds of *unit* arguments and then define several other argument notions consisting of these two *unit* types in different ways. We finally define various structures of argument in terms of the various definitions of argument types.

Definition 9. The types of arguments can be defined as follows:

- (1) An argument A is a unit I argument iff A has the form $B \Rightarrow \psi$ and subargument B is an atomic argument $B : \varphi$. We call the inference rule $\varphi \Rightarrow \psi$ a unit I inference.
- (2) An argument A is a unit II argument iff A has the form $B_1, \ldots, B_n \Rightarrow \psi$ and subarguments $A : B_1, \ldots, B_n$ are atomic arguments $B_1 : \varphi_1, \ldots, B_n : \varphi_n$. We call the inference rule $\varphi_1, \ldots, \varphi_n \Rightarrow \psi$ a unit II inference.
- (3) An argument A is a multiple unit I argument iff all inferences r_1, \ldots, r_n in the argument A are unit I inferences.
- (4) An argument A is a multiple unit II argument iff all inferences r_1, \ldots, r_n in the argument A are unit II inferences.
- (5) An argument A is a mixed argument iff A has at least one unit I subargument and unit II subargument.

We display the diagrams of argument types in Figure 3. For simplicity, we assume n = 2 in these diagrams and show only one case of a *mixed* argument.



Figure 3. Argument types

Proposition 1. Every argument is of exactly one argument type.

Proof. Firstly, we prove the existence of an argument type by induction on the number of unit inferences. For n = 1, argument A corresponds to a *unit* I argument. For n = k > 1, argument A corresponds to a *multiple unit* I argument, a *multiple unit* II argument, or a *mixed* argument. For n = k + 1, we represent argument A as $B_1, \ldots, B_n \Rightarrow \psi$, where $m \le n$. Consider the following possibilities:

- (1) If A_i is a multiple unit I argument and r_{k+1} is an unit I inference, then according to Definition 7 and Definition 9(3), A is a multiple unit I argument.
- (2) If A_i is a multiple unit I argument and r_{k+1} is an unit II inference, then according to Definition 7 and Definition 9(5), A is a mixed argument.
- (3) If A_i is a multiple unit II argument and r_{k+1} is an unit I inference, then according to Definition 7 and Definition 9(5), A is a mixed argument.
- (4) If A_i is a multiple unit II argument and r_{k+1} is an unit II inference, then according to Definition 7 and Definition 9(4), A is a multiple unit II argument.

(5) If A_i is a *mixed* argument and r_{k+1} is an *unit* I or *unit* II inference, then according to Definition 7 and Definition 9(5), A is a *mixed* argument.

Secondly, we prove the property of uniqueness of argument type. Assume there exists an argument A corresponding to two or more argument types: then there must exist two or more top rules in the argument, and then there are two or more conclusions in A, which contradicts the definition of argument.

Consider again Example 3. We have that A_1, A_2, A_3, A_4, A_5 are atomic arguments, A_8 is a *unit I* argument, A_6 is a *unit II* argument, A_9 is a *multiple unit I*, A_7 is a *multiple unit II*, and A_{10} is a *mixed* argument.

We next define several argument structures, which are sets of arguments with certain properties.

Definition 10. A set of arguments $\{A_1, \ldots, A_n\}$ is interconnected *iff for any argument* A_i *there exists an argument* A_j *such that* $Conc(A_i) \in Prem(A_j)$ *or* $Con(A_j) \in Prem(A_i)$ *or* $Con(A_i) = Con(A_j)$ *or* $Prem(A_i) = Prem(A_j)$.

Definition 11. The set of argument structures³ is defined as follows:

- (1) A set of arguments $\{A_1, \ldots, A_n\}$ is in a serial convergent structure SCS iff there are only unit I arguments in the set of arguments $\{A_1, \ldots, A_n\}$ and for any A_i and A_j we have $Conc(A_i) = Conc(A_j)$.
- (2) A set of arguments $\{A_1, \ldots, A_n\}$ is in a serial divergent structure SDS iff there are only unit I arguments in the set of arguments $\{A_1, \ldots, A_n\}$ and for any A_i and A_j we have $Prem(A_i) = Prem(A_j)$.
- (3) A set of arguments $\{A_1, \ldots, A_n\}$ is in a linked convergent structure LCS iff it contains only unit II arguments and for any A_i and A_j we have $Conc(A_i) = Conc(A_j)$.
- (4) A set of arguments $\{A_1, \ldots, A_n\}$ is in a linked divergent structure LDS iff it contains only unit II arguments and for any A_i and A_j we have $Prem(A_i) = Prem(A_j)$.
- (5) A set of arguments $\{A_1, \ldots, A_n\}$ is in a mixed structure MS iff it is interconnected and it is not of the form of either SCS, SDS, LCS and LDS.

We display the diagrams of argument structures in Figure 4. For simplicity, we assume n = 2 in the diagrams and show only one case of *mixed structure*.

Corollary 1. A set of arguments $S = \{A_1, \ldots, A_n\}$ is interconnected if for any $A_i \in S$, there is an argument A, such that A_i is a maximal proper subargument of A.

4.1 Reconsidering the standard approach

First, we consider the correspondence between the standard approach and our new approach. It is easy to see that single, linked and serial arguments, respectively, correspond to *unit I*, *unit II* and *multiple unit I* arguments.

However, *convergent* and *divergent* arguments are not arguments any more, since a *convergent* 'argument' now is an argument structure consisting of a number of distinct *unit I* arguments for the same conclusion, while a *divergent* 'argument' now is an argument structure consisting of a number of distinct *unit I* argument with the



Figure 4. Argument structures

same premise. For instance, in Example 1(1) there are two arguments $A \Rightarrow (C)$ and $B \Rightarrow (C)$ for the same conclusion (C), and in Example 1(2), there are two arguments $A \Rightarrow (B)$ and $A \Rightarrow (C)$ with the same premise (A) where but different conclusions.

Therefore, the classes of convergent, divergent 'arguments' are not arguments but argument *structures*. Actually, they correspond to the *serial convergent structure* SCS and the *serial divergent structure* SDS. Moreover, the class of complex arguments in the standard approach is not an argument if it contains SCS or SDS, but instead corresponds to the mixed argument structure MS. Otherwise, it corresponds to a mixed argument.

From the above analysis we see that the standard approach is incomplete and, moreover, does not distinguish types of individual argument from types of argument structures. We can conclude that the new classification in terms of the $ASPIC^+$ framework is helpful in clarifying and complementing the standard approach.

5 The problem of hybrid arguments

In this section we analyze why Vorobej's class of hybrid arguments is not needed if our approach is adopted. In our new approach, Vorobej's hybrid 'arguments' are not arguments but argument structures consisting a number of arguments. More specifically, they are of type *mixed structure* MS or *linked convergent structure* LCS.

We first make a notion explicit and redefine a definition. In [4] the notion of relevance is implicit and Vorobej treated it as a primitive dyadic relation. We note that there are two kinds of relevance: *defeasible relevance* indicates the support from a set of arguments to the conclusion via a defeasible inference, while *strict relevance* indicates the support form a set of arguments to the conclusion via a strict inference.

In the $ASPIC^+$ framework, we write $S \vdash \varphi$ if there exists a strict argument for φ with all premises taken from S, and $S \vdash \varphi$ if there exists a defeasible argument for φ with all premises taken from S. Then Definition 4 can be rewritten as follows:

Definition 12. A set of premises Σ supplements a set of premises Δ iff (1) $\Sigma \not\models C$ and $\Sigma \neq \emptyset$; (2) $\Delta \models C$; (3) $\Sigma \cup \Delta \vdash C$ or $\Sigma \cup \Delta \models C$, and (4) $\Sigma \cup \Delta$ is the minimal set satisfying clauses (1),(2) and (3) when $\Sigma \cup \Delta \vdash C$.

If a set of premises $\Sigma = \{P_1, \ldots, P_m\}$ supplements a set of premises $\Delta = \{Q_1, \ldots, Q_n\}$, then we have two arguments A

³ The structure here is different from the structure in informal approaches, where it refers to the structure of an *individual* argument.

and B, where argument A is of the form $Q_1, \ldots, Q_n \Rightarrow C$ and argument B is of the form $P_1, \ldots, P_m, Q_1, \ldots, Q_n \Rightarrow C$ or $P_1, \ldots, P_m, Q_1, \ldots, Q_n \rightarrow C$.

Thus, the hybrid argument here is a (1) mixed structure MS consisting of a unit I argument and a unit II argument, if m = 1, or (2) a linked convergent structure LCS consisting of two linked arguments, if m > 1.

We now first reconsider Example 2.

• (F): (1) All the ducks that I' ve seen on the pond are yellow. (2) I've seen all the ducks on the pond. (3) All the ducks on the pond are yellow.

Arguably, (1) supports (3) because of the defeasible inference rule of enumerative induction:

• All observed F's are G's \Rightarrow all F's are G's.

Moreover, (1) and (2) together arguably support (3) because of a deductive version of enumerative induction:

• All observed *F*'s are *G*'s, all observed *F*'s are all *F*'s → all *F*'s are *G*'s.

We then see that the apparently hybrid argument is in fact a convergent structure consisting of two separate arguments for the same conclusion, sharing one premise:

 $A = 1 \Rightarrow$ All the ducks on the pond are yellow. $B = 1, 2 \rightarrow$ All the ducks on the pond are yellow.

Actually, all examples in [4] can be reconstructed in terms of these two kinds of structures:

Example 4. Consider examples (G) and (J) as follows:

- (G): (1) My duck is yellow. (2) Almost without exception, yellow ducks are migratory. (3) My duck is no exception to any rule. (4) My duck migrates.
- (J): (1) Data quacks. (2) Data has webbed feet. (3) 95% of those creatures who both quack and have webbed feet are ducks. (4) Data is a duck.

In example (G), we have that $\{(1), (2)\} \vdash (4)$ and $\{(1), (2), (3)\} \vdash (4)$, so we have two arguments A and B for the same conclusion:

- $A = 1, 2 \Rightarrow (4)$ with a defeasible inference rule: almost without exception X's are Y's, a is an $X \Rightarrow a$ is a Y;

- $B = 1, 2, 3 \rightarrow (4)$ with a strict inference rule: almost without exception X's are Y's, a is a X, a is no exception to any rule $\rightarrow a$ is a Y.

In example (J), there are four arguments A, B, C and D based on $\{(1)\} \sim (4), \{(2)\} \sim (4), \{(1), (2)\} \sim (4) \text{ and } \{(1), (2), (3)\} \sim (4)$:

- $A = 1 \Rightarrow (4)$ with a defeasible inference rule: x quacks $\Rightarrow x$ is a duck;

- $B = 2 \Rightarrow (4)$ with a defeasible inference rule: x has webbed feet $\Rightarrow x$ is a duck;

- $C = 1, 2 \Rightarrow (4)$ with a defeasible inference rule that aggregates the two previous inference rules;

- $D = 1, 2, 3 \Rightarrow (4)$ with a defeasible inference rule: a is a Y, a is a

Z, 95% of x's who are both Y and Z are $T \Rightarrow a$ is a T.

On our account argument (G) is a *linked convergent structure* and argument (J) is a *mixed structure*.



Figure 5. Hybrid Arguments

6 Conclusion

In this paper we showed how AI models of argumentation can be used to clarify and extend informal-logic approaches to the structure of arguments. We defined a complete classification of types of arguments, we showed that convergent and divergent 'arguments' are not arguments but sets of arguments and we showed that Vorobeij's 'hybrid arguments' can be defined by explicitly distinguishing between deductive and defeasible inferences.

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