Counterexamples and Degrees of Support

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Abstract. My goal is to present recent work in the logic of counterexamples that could be of value to experts working to create computer models of arguments in natural language.

A very crucial skill in the evaluation of an argument in natural language (which I will also refer to as a "natural argument") is the construction of counterexamples to assess the support of its premises for its conclusion. So, if a computational model of natural argument neglected the construction and evaluation of counterexamples, then it would be very seriously deficient. To my knowledge there have not been any publications, besides my own single publication (see [3]), on the logic of counterexamples in natural language. Of course some argumentation and critical thinking textbooks mention counterexamples, but they offer superficial suggestions. Argumentation can be an odd discipline because it sometimes discovers what needs to be investigated *after* critical thinking textbooks have been published. This paper represents my attempt to further explore the logic of counterexamples in natural language. I will first contrast two different kinds of counterexamples, and then use one of them to assess the degrees of support of premises for their conclusion. Since I know nothing about computational models or artificial intelligence, and most of the members of my audience work in at least one of these areas, I will not be able to present my ideas in a way that is familiar to you. However, I will attempt to present my work as clearly as possible and make occasional references in the paper where I suspect that particular challenges would arise for those who would venture to construct computer models of counterexamples.

It is very easy to assess the *validity* of many everyday arguments: we simply construct a counterexample by imagining a situation where all the premises are true and the conclusion false. However, the standard use of this technique is inadequate against arguments that are not intended to be valid. Most everyday arguments are not intended to provide conclusive support. In other words, for most everyday arguments, if all their premises were true, their conclusions would be intended to be probably true, but not necessarily true. Given the general ease of inventing counterexamples against the validity of an argument, I will explore the logic of such counterexamples in order to find a way of using them to assess degrees of support that are less than conclusive.

Since there are two basic kinds of counterexamples against the validity of arguments, and my investigation will apply to only one of them, I will first clarify the distinction between them. An accurate computational model of natural argument would need to take these distinctions into account. The counterexamples whose logic I will be examining are very different from counterexamples by analogy. No textbook author describes in any detail how they differ, but only a few do present them as being different (see [1, 2, 4, 5]). We can see their differences by comparing and contrasting them when they are advanced against the same invalid argument. Let that argument be:

(A) (1) Derrida will pass the logic course only if he registers for the course.
(2) He has registered for the course.
So, (3) Derrida will pass the logic course.

This argument has the form, (1) P only if Q. (2) Q. So, (3) P. The fact that this is an example of the formal fallacy of affirming a consequent, and that we would typically quickly reject the argument without using any kind of counterexample, is irrelevant. I am just using it as an example against which both kinds of counterexamples can be advanced. Once we have identified the logical form of an argument, a counterexample by analogy against that argument must have the same form, but have true premises and a false conclusion. The more obviously true the premises and obviously false the conclusion, the more effective is the counterexample by analogy in showing the invalidity of a particular form. I suspect that this would be a challenge for computer models of natural arguments because what is obviously true and obviously false will vary according to the knowledge, intelligence, and experience of one's audience. In this particular case we can advance the following counterexample by analogy against argument (A):

CE1[2] against argument (A):
(1) There's a fire in this room only if there's oxygen in this room.
(2) There's oxygen in this room.
So, (3) there's fire in this room.

Let us now contrast it to the next counterexample:

CE2 against argument (A)
It is possible that:
(1) Derrida will pass the logic course only if he registers for the course. **AND**
(2) He has registered for the course. **AND**
What if Derrida does not do adequate studying. **AND**
Not-(3): *It is not the case* that Derrida will pass the logic course.

Differences

Both counterexamples successfully show that argument (A) is invalid, in other words, they both show that its premises are not suffi-

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[2] I will be using special notation to distinguish arguments and their counterexamples: "CE1 against argument (A)" simply means "counterexample number 1 against argument (A)", and "CE2 against argument (A)" means "counterexample number 2 against argument (A)".

cient for its conclusion. However, there are some logically significant differences between them[3]

1. A counterexample by analogy is an *argument* analogous in form to the argument against which it is advanced. But a counterexample such as CE2 is *not* an argument, and so such a counterexample cannot have the form of the argument against which they are advanced. A counterexample by analogy is *not* a mere conjunction of propositions. However, the kind of counterexample illustrated by CE2 is a mere possible conjunction of propositions. Accordingly, I propose that we name it a **"counterexample by possible conjunction"**. I invite anyone to propose a better descriptive label that will clearly differentiate this kind of counterexample from counterexamples by analogy.

2. In a counterexample by possible conjunction each premise of an argument is granted and unchanged (all the given reasons *as stated* are assumed to be true), and the argument's conclusion is negated. These two characteristics are necessary because the goal of a counterexample by possible conjunction is to show that all the given premises are not jointly sufficient for the truth of their conclusion. In contrast to these two characteristics, counterexamples by analogy, as illustrated by CE1, alter some of the content of the premises and conclusion, and they do not negate the conclusion.

3. In counterexamples by possible conjunction all the given premises of an argument and the negation of its conclusion are conjoined to a finite number of other statements, e.g. "What if Derrida does not do adequate studying" in CE2. These statements play the very important role of *making us understand how it is possible for all the given premises to be true and the conclusion false.*
   Why is this understanding so important? Though a counterexample by possible conjunction is not *in itself* an argument, it is evidence advanced to show to someone who has presented an argument that his/her premises are not sufficient. If a counterexample is not understood by the person presenting the argument, then s/he will not be convinced that the premises are not sufficient, in other words, s/he will not be convinced that his/her argument is invalid. Thus, understanding the counterexample, which involves understanding how it is possible for the argument's premises to be true and its conclusion false, is a necessary condition to show to an arguer that his/her argument is invalid. This is analogous to the construction of any argument: if the argument is not understood by its intended audience, then the it will not be convincing, even if it is impeccably logical and has necessarily true premises. This aspect of the construction of a counterexample is very context dependent: it will be effective generally only when it is sensitive to the level of knowledge, intelligence, and imagination of the person to whom the counterexample is presented. And these three factors affect one's level of understanding. It appears that computer models of natural arguments encounter again the challenge of context and audience dependence, but now there is the additional challenge of adequately representing the nebulous concept of understanding in computer models.
   Given this crucial role of the statements conjoined to the granted premises and negated conclusion to form a counterexample by possible conjunction, I need a convenient way to distinguish them from the premises and negated conclusion. I will thus sometimes label them by means of the letter "X". Since CE2 would typically be succinctly presented as "*What if* Derrida does not do adequate studying", and this common way of communicating this kind of counterexample focuses *exclusively* on the statements that make us understand how it is possible for all the given premises to be true and the conclusion false, I propose to name them the **"what-if-statements"** of the counterexample. Again, I invite anyone to propose a better label. In contrast to these counterexamples, no new statement is added to a counterexample by analogy.

4. The conjunction constituting the counterexample, $P_1 \& P_2 \ldots \& P_n \& X_1 \& X_2 .... \& X_n \& \tilde{} C$, is just presented as a logical possibility. However, as illustrated by CE1, a counterexample by analogy can have actually true premises and an actually false conclusion. I am wondering whether the notion of possibility can be easily represented in computer models of natural arguments. If not, there is another challenge here.

5. Counterexample CE2 has the specific form, *it is possible that* $P \& X \& \tilde{} C$. The general form of a counterexample by possible conjunction is, *it is possible that* $P_1 \& P_2 \ldots \& P_n \& X_1 \& X_2 \ldots \& X_n \& \tilde{} C$. Of course these conjuncts could be in any order, but I present them in this order because it is clearer, and because this order closely parallels the general structure of the argument against which it is advanced.
   In contrast, counterexamples by analogy do not have a common general logical form. For as illustrated by CE1, the form of a counterexample by analogy must correspond precisely to the form of the specific argument against which it is advanced, and of course there is no specific form common to all arguments. For example, not all arguments correspond in form to argument (A).

6. Counterexamples by possible conjunction help us to identify implicit assumptions of an argument. For example CE2 shows that argument (A) rests on the assumption that Derrida does or will do sufficient amount of studying. In other words, argument (A) assumes the *contradictory of the what-if-statement* in counterexample CE2. It must assume it in order to block counterexamples that use that specific what-if-statement. Such counterexamples are blocked because they must grant all the premises of the argument against which they are advanced; and if a reconstructed argument contains the negation of a what-if-statement as a premise, no counterexample can use that what-if-statement, and so such counterexamples are automatically eliminated. Counterexamples by analogy, on the other hand, do not identify any implicit assumptions. It seems that if a computer model could effectively construct counterexamples by possible conjunctions, it would be very easy to identify this kind of implicit assumption: it's simply the negation of the what-if statement.

7. The consequences of these two types of counterexamples are different. A successful counterexample by possible conjunction shows that the *specific premises*, $P_1 \& P_2 \ldots \& P_n$, are not sufficient for the truth of a *specific conclusion C*: these specific premises do not guarantee the truth of that specific conclusion. However, a successful counterexample by analogy shows that *the specific form* it expresses is invalid, and consequently, it proves that *any argument having its form* (and no other form that is valid)[4]

[3] The following seven points were presented at the Eleventh NCA/AFA Conference on Argumentation in August 1999, and published in the refereed proceedings of that conference (see [3]).

[4] I include this parenthetical phrase in order to take into account the fact that an argument can have more than one form, and is usually considered valid

is invalid. So, *no* premises of any argument having this form (and no other form that is valid) are sufficient for the truth of conclusion C.

From the preceding differences it follows that counterexamples by possible conjunction and by analogy are two very different kinds of counterexamples.

Consistency in Counterexamples by Possible Conjunction

I will next show that the *mere consistency* among the granted premise(s), the what-if-statement(s), and the negated conclusion in a counterexample by possible conjunction is not enough for the counterexample to show us that those premises are not sufficient for their conclusion. Consider the following counterexample against argument (B):

(B) (1) Winds are blowing a rain storm in our direction.
So, (C) it's going to rain here tomorrow.

CE3 against argument (B)
It is possible that:
(1) Winds are blowing a rainstorm in our direction. **AND**
What-if-statement: "Sirius" is the name of the closest star to our solar system. **AND**
Not-(C): It is not the case that it's going to rain here tomorrow.

The counterexample has the correct form, *it is possible that* $P\&X\&\tilde{}C$, and all the propositions are consistent, yet the counterexample fails to show us that the premise is not sufficient for the conclusion.

Contrast it to the next example:

CE4 against argument (B)
It is possible that:
(1) Winds are blowing a rainstorm in our direction. **AND**
What-if-statement: Strong winds from another direction are going to divert the storm away from us. **AND**
Not-(C) It is not the case that it's going to rain here tomorrow.

This counterexample is effective in proving to us that the premise is not sufficient for its conclusion. Since the only difference between counterexamples CE3 and CE4 is that it is only in the latter case that the what-if-statement makes us understand how it possible for the premise to be true and its conclusion false, then that understanding is a necessary condition for a counterexample to show us that premises are not sufficient for their conclusion. A discussion of the logic involved in making us understand how it is possible for premises to be true and their conclusion false is beyond the scope of this paper, and is not necessary in order to grasp the practical rudiments of this kind of counterexample. This particular logic will probably have to be well investigated if computer models of counterexamples are to be effective.

if it has at least one valid form. For example, the argument, "All philosophers are human. All humans are mortal. So all philosophers are mortal." has at least two forms. If we consider only the propositions, there is the invalid form, "P. Q. So, R". But if we consider the quantifiers within the propositions, there is the valid form "All A are B. All B are C. So all A are C." This argument is valid even though it also has an invalid form.

Counterexamples by possible conjunction and degrees of support

We have been examining some of the logic of the typical use of counterexamples by possible conjunction: to determine whether an argument is valid. Whenever a counterexample is successful, it proves that an argument's premises are not sufficient for (do not guarantee/necessitate) its conclusion. The serious limitation of this standard use is that the premises of most everyday arguments are not intended to be provide conclusive support, but rather only significant support. We will now explore a way to use these counterexamples to estimate the degree of support that is less than conclusive.

Elementary probability theory suggests a way to begin examining the logic of this additional role. I hope that my use of probability will also help you in your computer modeling of counterexamples.

(1) Pr($\tilde{}P$ or P) = 1.
(2) Pr($\tilde{}P$) + Pr(P) = 1.

We are looking for a substitution of "$\tilde{}P$" and "P" that will allow us to assess the degree of support of any argument, $P_1\&P_2\ldots\&P_n$, so C. Let the degree of support be expressed by the probability of C given $P_1\&P_2\ldots\&P_n$: $Pr(C \mid P_1\&P_2\ldots\&P_n)$. Replace both "$P$'s" in (2) by, $P_1\&P_2\ldots\&P_n\&\tilde{}C$:

(3) $Pr(\tilde{}(P_1\&P_2\ldots\&P_n\&\tilde{}C))+Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C) = 1.$

Subtract $Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C)$ from both sides of the equation:

(4) $Pr(\tilde{}(P_1\&P_2\ldots\&P_n\&\tilde{}C) = 1 - Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C).$

Replace $(P_1\&P_2\ldots\&P_n\&\tilde{}C)$ in (4) by the logically equivalent expression, $(P_1\&P_2\ldots\&P_n \Rightarrow C)$, which stands for the relation of support that the premises bring to the conclusion:

(5) $Pr(P_1\&P_2\ldots\&P_n \Rightarrow C) = 1 - Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C).$

The standard way of expressing the premises' support for the conclusion is rather:

(6) $Pr(C \mid P_1\&P_2\ldots\&P_n) = 1 - Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C).$

On the right side of this equation $Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C)$ represents the probability of all the counterexamples by possible conjunction against the argument $P_1\&P_2\ldots\&P_n$, so C, whose support is represented on the left side of the equation.

Formula (6) coincides with our intuitions. First, when there are no counterexamples, the formula derives what we would expect with a deductively valid argument, for when $Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C) = 0$, then $Pr(C \mid P_1\&P_2\ldots\&P_n) = 1$: if the premises were true, the conclusion would also be true. Secondly, it entails that the greater the probability of all those counterexamples, the weaker the support (i.e. the smaller the probability of the conclusion given that all its premises are true), and the smaller the probability of all the counterexamples, the stronger the support for the conclusion. There is thus an inverse relation between the probability of the counterexamples and the strength of the support (the probability of the conclusion when all its premises are true). This inverse relation seems to be an aspect of this extended use of counterexamples that could be easily implemented in a computer model of natural arguments. But now we move to greater challenges.

How do we *estimate* the probability of $Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C)$? Let us examine an everyday argument and various counterexamples against it.

C (1) Each student beginning my course is sufficiently intelligent to pass the course.
(2) So, each student beginning my course will pass it.

CE5 against argument (C)
It is possible that:
P: Each student beginning my course is sufficiently intelligent to pass the course. **AND**
X: What if at least one student will be sick too often to do all the necessary work to pass. **AND**
$\tilde{}C$: It is not the case that each student beginning my course will pass it.

Consider the following condensed counterexamples against argument (C). Assume that their what-if-statements, represented by "X", are conjoined to $P\&\tilde{}C$, and that the conjunction of all these statements forming each counterexample falls within the scope of the operator, "it is possible that", just as in CE5 against argument (C).

CE6(C) X: What if at least one students will not study material that must be studied to pass it.

CE7(C) X: What if at least one student has family responsibilities that very seriously interfere with his/her academic performance.

CE8(C) X: At least one student has personal problems that very seriously interfere with his/her academic performance.

CE9(C) X: What if the teacher will grade unfairly.

CE10(C) X_1: What if there is a personality conflict between the teacher and at least one student. **AND**
X_2: What if that student drops the course.

Regardless of the actual probability of any specific counterexample by possible conjunction, it is significantly smaller than the Pr(CE5(C) **or** CE6(C) **or** CE7(C) **or** CE8(C) **or** CE9(C) **or** CE10(C)). So, if we were to use the probability of only one counterexample to estimate the degree of support, and discard the probability of this disjunction of counterexamples, then we would significantly overestimate the degree of support of the premise - even if the chosen counterexample had the highest probability. Each counterexample must be included in our estimation of the degree of support because each one exposes other serious weaknesses in the support that would be overlooked even by the most probable counterexample. Since most everyday arguments are vulnerable to more than one counterexample with probabilities worth considering, we must take into account not just the most probable counterexample but also other probable counterexamples.

Hence, formula (6) can be restated more precisely as:

(7) $Pr(C \mid P_1\&P_2\ldots\&P_n) = 1 - Pr(\text{CE1orCE2orCE3...orCEn})$.

(I will address one of the challenges of estimating such a disjunction of probabilities later.) However, this added formulaic precision does not necessarily give us an accurate degree of the support of premises, for we very rarely have all the counterexamples against the support an argument, and consequently our estimation of the support is very rarely final and complete. This is a challenge not just for computer models of natural argument, but for anyone who wants a rough estimation of the degree of support.

Formula (7) can be further simplified. In any counterexample by possible conjunction all the given premises and the negation of the conclusion are assumed to be true:

$$Pr(P_1\&P_2\ldots\&P_n) = Pr(\tilde{}C) = 1.$$

Since the probability of a typical counterexample is,

$$Pr(P_1\&P_2\ldots\&P_n\&X_1\&X_2\ldots X_n\&\tilde{}C) = Pr(P_1\&P_2\ldots\&P_n)xPr(X_1\&X_2\ldots X_n)xPr(\tilde{}C),$$

then

$$Pr(P_1\&P_2\ldots\&P_n\&X_1\&X_2\ldots X_n\&\tilde{}C) = Pr(X_1\&X_2\ldots X_n).$$

Hence, when talking about the probability of a counterexample by possible conjunction, we are talking about the probability of the conjunction of its what-if-statements. Therefore, (7), which includes more than one counterexample against a the support of an argument, can be more simply formulated as:

(8) $Pr(C \mid P_1\&P_2\ldots\&P_n) = 1 - Pr(X1_1\&X1_2\ldots\&X1_n or X2_1\&X2_2\ldots\&X2_n\ldots or Xn_1\&Xn_2\ldots\&Xn_n)$

There is a further challenge for natural language users and computer experts to meet when using counterexamples by possible conjunction to estimate the degree of support of premises: they must determine when to stop constructing counterexamples. For instance, we could have continued inventing more counterexamples against the support of argument (C). If we wanted to have a reliable estimate of the degree of support that (C)'s premises give to its conclusion, where should we stop? Assuming that time is not an obstacle, we stop when we can only invent extremely unlikely counterexamples, and we have reason to believe that we would continue inventing only such unlikely ones. Here is an example of an extremely improbable counterexample:

CE11 against argument (C)
It is possible that:
P: Each student beginning my course is sufficiently intelligent to pass the course. **AND**
X: What if at least one student is abducted by an extraterrestrial at the beginning of the course. **AND**
$\tilde{}C$: It is *not* the case that each student beginning my course will pass it.

We stop when we can construct only very unlikely counterexamples because they add nothing significant to the probability of the disjunction of all the realistic counterexamples we have already constructed. It is important to bear in mind that wherever we stop, it will be due to our limited knowledge and imagination. So, we can *never* be sure that we have taken into consideration *all* the counterexamples that are represented by $Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C)$ in formula (6) $Pr(C \mid P_1\&P_2\ldots\&P_n) = 1 - Pr(P_1\&P_2\ldots\&P_n\&\tilde{}C)$. For this reason, it is sometimes important to persist inventing a few the wildly imaginative counterexamples because sometimes that process can help us to discover more realistic ones.

There is a further practical challenge in determining $Pr(X1_1\&X1_2\ldots\&X1_n or X2_1\&X2_2\ldots\&X2_n\ldots or Xn_1\& Xn_2\ldots\&Xn_n)$: not all counterexamples (or more simply, not all what-it-statements) are independent of one another. Event M is independent of event N if and only if N does not affect the probability of M: if and only if Pr(M, given N) = Pr(M). For instance, if I am boarding a taxi for a destination that is five miles away, and I infer from my taking the taxi that I will arrive at my destination in less than an hour, there are many interdependent counterexamples against the inference: what if there is an accident; what if there is a flat tires; what if the driver becomes sick. These are different physical possibilities that could prevent me from reaching my destination on time, and they are partly interdependent: some accidents are caused by flat tires, and some accidents are caused by a driver's illness. If we are to continue using probability theory, then matters would get complicated. For if events M and N are not independent, then Pr(M or N) = Pr(M) + Pr(N) - Pr(M & N), and so the estimation of the probability of my arriving on time would need to include the probability of a flat or an accident, which equals the sum of the probability of a flat and the probability of an accident, minus the probability of the conjunction of a tire having a flat and the taxi driver having an accident. Given the interdependence of many daily counterexamples, the costs, in terms of time, mental energy, and possibly even money, of this further application of probability theory would seem to outweigh the benefits.

How would we estimate the probability of the counterexamples against argument (C) if we also estimated their interdependence? My estimation of the probability of the disjunction of those six counterexamples, without considering their interdependence, is that it is *at least* moderately probable. Consequently, the probability of the conclusion that each student beginning my course will pass it is *at most* moderately improbable. What would I change if I now take into consideration the interdependence of the counterexamples? The overall combined probability of all the counterexamples would have to diminish, and there would be a corresponding increase in the strength of the support. What would be the amount of that change? My estimation is that the probability of the conjunction of all those counterexamples would still be roughly at least moderately probable. So my consideration of the interdependence makes me only qualify my estimation with "roughly".

If this ordinary example is representative of most everyday examples, then for practical everyday purposes, will considerations of the interdependence of counterexamples be useful?

In most situations we don't have the information or the time to figure out Pr(M & N), it is challenging enough just to estimate Pr(M) and Pr(N). However, knowing that some counterexamples against the support of an argument are interdependent makes us aware that the disjunction of the counterexamples' probabilities is in fact less than the sum of their individual probabilities, thereby indicating from the inverse relation that the support of the premises is stronger than initially estimated. The greater the interdependence between counterexamples (i.e. the greater the probability of one given the other) against the support of an argument, and the greater the number of interdependent counterexamples, the smaller the sum of the counterexamples' individual probabilities; and consequently, the stronger the support of the argument's premises. It is possible that in some cases, the interdependence might be significant and easy to estimate, thus we might easily realize the significant decrease of the probability of a disjunction of counterexamples against an argument. Though for most everyday purposes these consideration will be beyond our knowledge and available time, it might be prudent in some cases to raise the questions, "Are there any interdependent counterexamples? To what degree are they interdependent?". By realizing the extent of interdependence and the number of the counterexamples, we come to see that the degree of support of premises is stronger than what we might have initially estimated. In order not to underestimate the degree of support that premises bring to their conclusion, it might be prudent in some cases to raise the question, "How many counterexamples are interdependent? To what degree are they interdependent?".

In this paper I identified two kinds of counterexamples: counterexamples by possible conjunction and counterexamples by analogy; described the logical differences between them; examined some of the logic of counterexamples by possible conjunction. It is a logically possible conjunction of all the premises of an argument (all are assumed true), the conclusion is negated, and one or more statements, named "what-if-statements". I showed that the latter have the very special function of making the proponent of an argument understand how it is possible for his/her premises to be true and conclusion false; argued that the mere consistency of all the statements constituting these counterexamples is not sufficient for the success of these counterexamples. I used elementary probability theory to justify extending the use of these counterexample to estimate the degree of premise support that is less than conclusive; showed that the strength of support (i.e. the probability of a conclusion given its premises) is inversely proportional to the probability of the disjunction of all the what-if-statements of successful counterexamples against the support; described where we should stop in the construction of counterexamples; illustrated some of the practical limitation of considering the interdependence of what-if-statements when estimating the probability of their disjunction.

If computer models of arguments in natural language are to be successful, they must be able to model all the natural and effective ways of assessing the support of premises. The construction of counterexamples by possible conjunction is a natural and effective way of assessing the sufficiency of premises (i.e., assessing the validity of an argument), and they can be used to estimate the degree of support that is less than conclusive. So, computer models of arguments in natural language should attempt to model the construction, use, and evaluation of these counterexamples. Therefore, programmers face the following challenges:

1. The models must distinguish counterexamples by possible conjunction and counterexamples by analogy.
2. The models must represent the concept of possibility.
3. The models must identify effective what-if statements of counterexamples by possible conjunction against an argument. This identification will depend on the models' ability to determine whether the what-if statements make the proponents of the argument understand how it is possible for all their premises to be true and their conclusion false. So the models must (a) handle the nebulous concept of understanding. They must also (b) be very context sensitive, for the understanding of an audience varies according to its knowledge, experience, and imagination.
4. The models must estimate the probability of the effective what-if statements of each counterexample, and estimate the disjunction of all the probabilities of the effective what-if statements against the same argument.
5. The models must determine when it is appropriate to consider the dependence among effective what-if statements, and how their dependence affects the combined probability of all the effective counterexamples against an argument.
6. The models must determine when it is appropriate to stop constructing counterexamples by possible conjunction.

Since I do not want to discourage any of you from investigating the modeling of these counterexamples, I would like to end by identifying two areas where programmers would probably *not* face any serious challenges:

7. It will be easy for models to identify certain kinds of implicit assumptions. For once an effective what-if statement of a counterexample is identified, it follows that the argument against which the counterexample is advanced assumes the negation of that what-if statement.
8. At a certain stage it will be easy to estimate the degree of support of premises. For when what-if statements are independent, or when their dependence is insignificant, the probability of a conclusion is simply 1 minus the estimated combined probability of the effective what-if statement of each counterexample.

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