A unified setting for inference and decision: An argumentation-based approach

Leila Amgoud
IRIT - CNRS
118, route de narbonne
31062, Toulouse, France
amgoud@irit.fr

Abstract

Inferring from inconsistency and making decisions are two problems which have always been treated separately by researchers in AI.
The aim of this paper is to present a *general* argumentation framework in which they are both captured. That framework can be used for decision under uncertainty, multiple criteria decision, rule-based decision and case-based decision. Moreover, works on classical decision suppose that the information about the environment is coherent, and this is no longer required by this general framework.

1 Introduction

Decision making and inference have been studied for a long time separately, and have been considered as two distinct problems. The basic idea behind inference is to make "safe" conclusions from a set of premises, whereas the decision making problem consists of selecting the "best" decision among different alternatives on the basis of the available information (the beliefs about the environment, the goals, etc.). In this paper we argue that inference is part of a decision process. The basic idea is to infer from all the available information, the formulae which are "correctly" supported, then to order the different decisions only on the basis of these formulae. Thus, a decision problem can be seen as a two steps process: i) inferring from inconsistency then ii) ordering the alternatives using any criterion among those defined in classical decision theory [5; 7; 6; 9].
We propose a general argumentation framework in which the two problems are analyzed and handled. This framework extends the one developed in [2] for inference. Argumentation is a reasoning model based on the construction of arguments in favor and against a given statement and then to select the most acceptable of them. Such an approach has indeed some obvious benefits, in particular, it is more acute with the way humans often deliberate and finally make a choice. Another feature of the proposed framework is that it extends classical work on decision theory in the sense that the hypothesis that the information about the environment is coherent is no longer required by this general framework. Moreover, the framework is general enough to capture different kinds of decision problems, namely, decision under uncertainty, multiple criteria decision and rule-based decision.

2 Argumentation process

Argumentation is a reasoning model which follows the following steps: constructing arguments (in *favor* of / *against* a "statement") from bases, defining the strengths of those arguments, determining the different conflicts between the arguments, evaluating the acceptability of the different arguments, and concluding. What distinguishes an argumentation framework for reasoning about beliefs and an argumentation framework for decision making is mainly the last step of the argumentation process. Indeed, in inference systems, consequence relations are defined in order to decide which conclusion should be inferred from a knowledge base. Those conclusions are considered "true". However, things seem different with decision making. The basic idea behind a decision problem is to define an ordering $\rhd$, on a set $\mathcal{D}$ of possible decisions on the basis of their supporting arguments.

In what follows, $\mathcal{L}$ denotes a logical language closed under negation. From $\mathcal{L}$, two kinds of rules can be built: *strict* rules and *defeasible* rules. Strict rules enable us to define conclusive inferences, whereas *defeasible* rules enable us to define only defeasible inferences.

Definition 1 (Strict/Defeasible rules) *Let $\mathcal{L}$ be a logical language. A* strict *rule is of the form $\phi_1, \ldots, \phi_n \rightarrow \phi$ meaning that if $\phi_1, \ldots, \phi_n$ are true, it is "always" the case that ϕ is true as well. A* defeasible *rule is of the form $\phi_1, \ldots, \phi_n \Rightarrow \phi$ meaning that if $\phi_1, \ldots, \phi_n$ are true, ϕ is "generally" true as well, where $\phi_1, \ldots, \phi_n, \phi \in \mathcal{L}$. $\mathcal{S}$ is the set of strict rules and $\mathcal{NS}$ is the set of defeasible rules. Let $\mathcal{R} = (\mathcal{S}, \mathcal{NS})$.*

From $\mathcal{L}$, four sets can be distinguished: 1) a set $\mathcal{D}$ which contains all the possible *decisions*, 2) a set $\mathcal{K}$ which contains the *beliefs* of an agent, 3) a set $\mathcal{G}^+$ which will gather the *positive goals* of an agent (a positive goal represents what an agent wants to achieve), and 4) a set $\mathcal{G}^-$ which will gather the *negative goals* of an agent. A negative goal represents what an agent rejects. In what follows, $\mathcal{T} = <\mathcal{R}, \mathcal{D}, \mathcal{K}, \mathcal{G}^+, \mathcal{G}^->$ will be called a *theory*.

3 The arguments

Two categories of arguments will be defined: *epistemic* arguments for supporting beliefs and *non-epistemic* arguments for supporting decisions. Among non-epistemic arguments, one may distinguish between *recommending* arguments and *decision* arguments. The idea is that a given decision may be justified in two ways: i) it is recommended in a given situation (in the case of *rule-based decision*), or ii) it satisfies / violates some goals of the decision maker (in the case of decision under uncertainty and in multiple criteria decision [4; 1]). Recommending arguments have a *deductive* form, whereas decision arguments have an *abductive* form. In what follows, for a given argument, the function PROP returns all the propositions used in that argument, CONC returns its conclusion and SUB returns all its sub-arguments.
The basic idea behind an epistemic argument is the fact that a given premise is justified on the basis of the available beliefs in $\mathcal{K}$. Note that goals are not used in the definition of epistemic arguments in order to avoid any *wishful thinking*.

Definition 2 (Epistemic Argument) *An* epistemic *argument* A *is: 1)* ϕ *if* $\phi \in \mathcal{K}$ *with:* $\texttt{PROP}(A) = \{\phi\}$, $\texttt{CONC}(A) = \phi$, $\texttt{SUB}(A) = \phi$,
2) $A_1, \ldots A_n \rightarrow \psi$ *(resp.* $A_1, \ldots A_n \Rightarrow \psi$*) if* $A_1, \ldots, A_n$ *are epistemic arguments such that there exits a strict rule* $\texttt{CONC}(A_1), \ldots, \texttt{CONC}(A_n) \rightarrow \psi$ *(resp. there exits a defeasible rule* $\texttt{CONC}(A_1), \ldots, \texttt{CONC}(A_n) \Rightarrow \psi$*) and* $\psi \in \mathcal{K}$. $\texttt{PROP}(A) = \texttt{PROP}(A_1) \cup \ldots \cup \texttt{PROP}(A_n) \cup \{\psi\}$, $\texttt{CONC}(A) = \psi$, $\texttt{SUB}(A) = \texttt{SUB}(A_1) \cup \ldots \cup \texttt{SUB}(A_n) \cup \{A\}$.
Let $\mathcal{A}_e$ *be the set of all epistemic arguments.*

In rule-based decision making, one generally gives when a decision can be applied. Such rules are captured in this framework in terms of strict or defeasible rules of the form $\phi_1, \ldots \phi_n \rightarrow d$ (resp. $\phi_1, \ldots \phi_n \Rightarrow d$) with $\phi_1, \ldots \phi_n \in \mathcal{K}$ and $d \in \mathcal{D}$. These rules mean that if $\phi_1, \ldots \phi_n$ are true then one "should" take the decision d (resp. "can" take the decision d). Recommending arguments are thus based on epistemic ones. Formally:

Definition 3 (Recommending Argument) *A* recommending *argument is* $A_1, \ldots A_n \rightarrow d$ *(resp.* $A_1, \ldots A_n \Rightarrow d$*) if* $A_1, \ldots, A_n$ *are epistemic arguments,* $d \in \mathcal{D}$ *and there exists a strict rule* $\texttt{CONC}(A_1), \ldots, \texttt{CONC}(A_n) \rightarrow d$ *(resp. a defeasible rule* $\texttt{CONC}(A_1), \ldots, \texttt{CONC}(A_n) \Rightarrow d$*).* $\texttt{PROP}(A) = \texttt{PROP}(A_1) \cup \ldots \cup \texttt{PROP}(A_n) \cup \{d\}$, $\texttt{CONC}(A) = d$, $\texttt{SUB}(A) = \texttt{SUB}(A_1) \cup \ldots \cup \texttt{SUB}(A_n) \cup \{A\}$.
Let $\mathcal{A}_r$ *be the set of all recommended arguments.*

In decision under uncertainty (resp. in multiple criteria decision), the preferred decisions are generally the ones which satisfy highly the goals/preferences (resp. the criteria). As shown in [4; 1], a decision is related to the goals via rules of the form $\phi_1, \ldots \phi_n, d \rightarrow \psi$ (resp. $\phi_1, \ldots \phi_n, d \Rightarrow \psi$) meaning that in the case where $\phi_1, \ldots \phi_n$ are true, if the decision d is taken then this leads to the satisfaction of the goal ψ.

Definition 4 (Decision Argument) *A* decision *argument is* $A_1, \ldots A_n, d \rightarrow \psi$ *(resp.* $A_1, \ldots A_n, d \Rightarrow \psi$*) if* $A_1, \ldots, A_n$ *are epistemic arguments and there exits a strict rule* $\texttt{CONC}(A_1), \ldots, \texttt{CONC}(A_n), d \rightarrow \psi$ *(resp. a defeasible rule* $\texttt{CONC}(A_1), \ldots, \texttt{CONC}(A_n), d \Rightarrow \psi$*) such that* $d \in \mathcal{D}$ *and* $\psi \in \mathcal{G}^+$ *(resp.* $\psi \in \mathcal{G}^-$*).* $\texttt{PROP}(A) = \texttt{PROP}(A_1) \cup \ldots \cup \texttt{PROP}(A_n) \cup \{\psi, d\}$, $\texttt{GOALS}^+(A) = \{\psi\}$ *(resp.* $\texttt{GOALS}^-(A) = \{\psi\}$*),* $\texttt{CONC}(A) = d$, $\texttt{SUB}(A) = \texttt{SUB}(A_1) \cup \ldots \cup \texttt{SUB}(A_n) \cup \{A\}$.
Let $\mathcal{A}_d$ *be the set of all decision arguments.*

Let $\mathcal{A} = \mathcal{A}_e \cup \mathcal{A}_r \cup \mathcal{A}_d$. Note that all the sub-arguments of a recommending argument (resp. a decision argument) are epistemic ones. Formally:

Property 1 *Let* $A \in \mathcal{A}_d$ *(resp.* $\mathcal{A}_r$*).* $\forall A' \in SUB(A)$ *such that* $A \neq A'$*, then* A' *is an epistemic argument.*

Unlike beliefs, a given decision may have an argument *in favor* of it and also an argument *against* it which are not necessarily conflicting. Intuitively, an argument is in favor of a decision if that decision leads to the satisfaction of a positive goal. The arguments which recommend decisions are also considered in favor of that decision. An argument is against a decision if the decision leads to the satisfaction of a negative goal. Hence, arguments PRO a decision stress the *positive consequences* of the decision, while arguments CONS are only focusing on the negative ones. Let's define two functions which return respectively for a given decision the arguments which are in favor of it and the arguments against it.

Definition 5 (Arguments PRO) *Let* $d \in \mathcal{D}$ *and* $B \subseteq \mathcal{A}$. $Arg_P(d, B) = \{A \in B \mid \texttt{CONC}(A) = d$ *and (*$\texttt{GOALS}^+(A) \neq \emptyset$, *or* $d \in \texttt{PROP}(A)$*)*$\}$.

Definition 6 (Arguments CONS) *Let* $d \in \mathcal{D}$. $Arg_C(d, B) = \{A \in B \mid \texttt{CONC}(A) = d$ *and* $\texttt{GOALS}^-(A) \neq \emptyset\}$.

4 Comparing arguments

In [3; 10], it has been argued that arguments may have forces of various strengths. Generally, the force of an argument can rely on the information from which it is constructed. Epistemic arguments involve only one kind of information: the *beliefs*. Thus, the arguments using, for instance, more certain beliefs are found stronger than arguments using less certain beliefs. Unlike epistemic arguments, arguments supporting decisions involve both *goals* and *beliefs*. Thus, the force of such arguments may depend not only on the quality of the beliefs used in these arguments, but also on the *importance* of the *satisfied* (resp. *violated*) goals. Note that since recommending arguments involve only beliefs, then their force may be defined in the same way as for epistemic arguments. see [1; 4] for some definitions of the argument's force.

The forces of arguments will play two roles: i) they allow an agent to compare pairs of arguments in order to select the 'best' ones, ii) they are useful for determining the acceptable arguments among the conflicting ones. In what follows $\succeq$ will denote any preference relation between arguments.

Notation 1 *Let* A, $B \in \mathcal{A}$. *If* $\succeq$ *is a pre-ordering, then* $A \succeq B$ *means that* A *is at least as 'good' as* B. $\succ$ *and* $\approx$ *will denote respectively the strict ordering and the relation of equivalence associated with the preference between arguments.*

Since an argumentation framework for decision making may have three categories of arguments: epistemic arguments, recommending arguments and decision arguments, one may show how mixed arguments can be compared, and how arguments of the same category may be compared. In [1; 4] different criteria for comparing arguments of the same nature have been defined. in this section we focus only on the comparison of mixed arguments.
Epistemic arguments always take precedence over arguments for decisions. The reason is that a decision cannot be well supported if the beliefs on which it is based are not justified.

Definition 7 (Epistemic vs arguments for decisions) *Let $A \in \mathcal{A}_e$, and $B \in \mathcal{A}_d$ (resp. $\in \mathcal{A}_r$). It always holds that $A \succ B$.*

In normative systems, for instance, where recommending arguments are built from laws and obligations, it is natural to prefer a recommending argument to a decision argument.

Definition 8 (Recommending vs decision arguments) *Let $A \in \mathcal{A}_r$, and $B \in \mathcal{A}_d$. It holds that $A \succ B$.*

5 Dialectical interactions between arguments

Since the information in $\mathcal{K}$, $\mathcal{D}$, $\mathcal{G}^+$ and $\mathcal{G}^-$ may be inconsistent, the arguments may be conflicting. Indeed, arguments supporting beliefs may be conflicting. It may also be the case that arguments supporting beliefs conflict with arguments supporting decisions. Finally, arguments supporting decisions can conflict with each other. Two different kinds of conflicts may exist between arguments of the same category and arguments of different categories. The first one is the rebutting.

Definition 9 (Rebutting) *Let A and B be arguments in $\mathcal{A}$.*
A rebuts *B iff $\exists \phi$ such that $\phi \in \texttt{PROP}(A)$ and $\neg\phi \in \texttt{PROP}(B)$. A* rebut-defeats *B iff A rebut-attacks B and not ($B \succ A$).*

Notice that rebut-attacks are symmetric.

Definition 10 (Undercutting) *Let A and B be arguments in $\mathcal{A}$.*
A undercuts and undercut-defeats B iff B has a subargument B' of the form $B''_1, \ldots, B''_n \Rightarrow \psi$ and A has a subargument A' with $\texttt{CONC}(A') = \neg\lceil\texttt{CONC}(B''_1), \ldots, \texttt{CONC}(B''_n) \Rightarrow \psi\rceil$.

The two above relations are brought together in a unique definition of "defeat".

Definition 11 (Defeating) *Let A and B be arguments. We say that A* defeats *B iff:*

- *A rebut-defeats B, or*
- *A undercut-defeats B.*

Note that since epistemic arguments are always preferred to decision and recommended arguments, then an epistemic argument cannot be defeated w.r.t *defeat* by a decision argument or a recommended argument.

Property 2 *It cannot be the case that $\exists\ A \in A_e$ and $\exists\ B \in A_d$ (or $B \in A_r$) such that B defeats A.*

6 Argumentation framework

Once all the basic concepts introduced, we are now ready to define an argumentation framework.

Definition 12 (Argumentation framework) *Let $\mathcal{T}$ be a theory. An* argumentation framework *(AF) built on $\mathcal{T}$ is a triple $<\mathcal{A}$,* defeat*, $\succeq>$.*

Among all the conflicting arguments, it is important to know which are the arguments which will be kept for inferring conclusions and for ordering decisions. In [8], different semantics for the notion of acceptability have been proposed: *grounded*, *preferred*, *stable* and *complete* semantics. Let's recall them here.

Definition 13 (Conflict-free, Defence) *Let $S \subseteq \mathcal{A}$.*

- *A set S is* conflict-free *iff there exist no A_i, A_j in S such that A_i defeats A_j.*
- *A set S* defends *an argument A_i iff for each argument $B \in \mathcal{A}$, if B defeats A_i there exists C in S such that C defeats B.*

Definition 14 (Acceptability semantics) *Let S be a subset of $\mathcal{A}$.*

- Admissible*: S is an admissible set iff S is conflict-free and S defends collectively all its elements.*
- Preferred*: S is a preferred extension iff S is maximal for set inclusion among the admissible sets of $\mathcal{A}$.*
- Complete*: an admissible subset S of $\mathcal{A}$ is a complete extension iff every argument which is defended collectively by S belongs to S.*
- Stable*: a subset S of $\mathcal{A}$ is a stable extension iff S is conflict-free and S defeats each argument which does not belong to S.*
- Grounded*: S is the grounded extension iff S is conflict-free and S is the least fixed point of the characteristic function F of $<\mathcal{A}, \mathcal{R}>$ (F: $2^{\mathcal{A}} \rightarrow 2^{\mathcal{A}}$ with $F(S) = \{A$ such that A is defended collectively by $S\}$).*

Let $\mathcal{S} = \{E_1, \ldots, E_n\}$ be the set of all possible extensions under a given semantics.

The above extensions may contain epistemic and non-epistemic arguments. Moreover, each argument which is in an extension, have all its sub-arguments in that extension.

Proposition 1 *Let $AF = <\mathcal{A}$,* defeat*, $\succeq>$ be an argumentation framework and $E_i \in \mathcal{S}$.*
$\forall\ A \in E_i$, $\texttt{SUB}(A) \in E_i$.

Once the acceptable arguments defined, the decisions may be compared on the basis of the quality of their supporting arguments, and conclusions may be inferred from a knowledge base.

Definition 15 (Inferring) *Let $AF = <\mathcal{A}$,* defeat*, $\succeq>$ be an argumentation framework.*
ψ is inferred *from $\mathcal{K}$, denoted by $\mathcal{K} \mid\!\sim \psi$, iff $\forall\ E_i \in \mathcal{S}$, $\exists\ A \in E_i \cap \mathcal{A}_e$ such that $\texttt{CONC}(A) = \psi$. $\texttt{Output}(AF) = \{\psi \mid \mathcal{K} \mid\!\sim \psi\}$.*

Elements of $\mathtt{Output}(AF)$ are considered as true. Note that decisions are not inferred. The reason is that one cannot say that a given decision is true or false. A decision may have only acceptable arguments which are against it. In such a situation that decision should be discarded. So, the idea in a decision problem, is to construct the arguments in favor and against each decision. Then among all those argument, only the strong (acceptable) ones are kept and the different decisions are compared on the basis of them.

Comparing decisions is an important step in a decision process. Below we present an example of an *intuitive* principle which is reminiscent of classical principles in decision.

Definition 16 (Comparing decisions) *Let* AF = $<\mathcal{A}$, defeat, $\succeq>$ *be an argumentation framework and* E *its grounded semantics. Let* d_1, $d_2 \in \mathcal{D}$.
Let $Arg_P(d_1, E)$ = (P_1, …, P_r) *and*
$Arg_P(d_2, E)$ = (P'_1, …, P'_s).
Each of these vectors is assumed to be decreasingly ordered w.r.t $\succeq$ *(e.g.* $P_1 \succeq \ldots \succeq P_r$*). Let* v = *min(*r, s*).*
A pre-ordering $\rhd$ *on* $\mathcal{D}$ *is defined as follows:* $d_1 \rhd d_2$ *iff:*

- $P_1 \succ P'_1$, *or*
- $\exists\, k \leq v$ *such that* $P_k \succ P'_k$ *and* $\forall\, j < k$, $P_j \approx P'_j$, *or*
- $r > v$ *and* $\forall\, j \leq v$, $P_j \approx P'_j$.

The above principle takes into account only the arguments pro, and prefers a decision which has at least one acceptable argument pro which is preferred (or stronger) to any acceptable argument pro the other decision. When the strongest arguments in favor of d_1 and d_2 have equivalent strengths (in the sense of $\approx$), these arguments are ignored.

We can show that modeling decision making and inference in the same framework does not affect the result of inference. Before that, let's define when two argumentation frameworks are equivalent.

Definition 17 (Equivalent frameworks) *Let (*$\mathcal{R}$, $\mathcal{D}$, $\mathcal{K}$, $\mathcal{G}^+$, $\mathcal{G}^-$*) be a theory.*
An argumentation framework AF = $<\mathcal{A}$, defeat, $\succeq>$ *is* equivalent *to another argumentation framework AF'* = $<\mathcal{A}'$, defeat', $\succeq'>$ *iff:* $\mathtt{Output}(AF)$ = $\mathtt{Output}(AF')$, *or the pre-ordering* $\rhd$ *is equivalent to* $\rhd'$, *i.e. for any decisions* $d, d' \in \mathcal{D}$, *if* $d \rhd d'$ *then* $d \rhd' d'$.

Proposition 2 *The two argumentation frameworks* $<\mathcal{A}_e$, defeat, $\succeq>$ *and* $<\mathcal{A}$, defeat, $\succeq>$ *are equivalent.*

The above proposition means that an argumentation framework in which only epistemic arguments are taken into account will return exactly the same result as a framework in which all the kinds of arguments are considered.

7 Conclusion

In this paper we have presented a general formal framework for decision making and inference. This offers for the first time a coherent setting for argumentation-based inference and decision. Unlike inference framework where only epistemic arguments exist, in a decision framework two categories of arguments can be built: epistemic ones and arguments for decisions. This is not surprising since decisions are based on some available knowledge. The basic idea behind a decision problem is to infer from the knowledge base justified conclusions which may support decisions if any. Then, decisions will be compared on the basis of the strengths of the arguments in favor and against them.

References

[1] L. Amgoud, J.-F. Bonnefon, and H. Prade. An argumentation-based approach to multiple criteria decision. In *Proceedings of the 8th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, 2005.

[2] L. Amgoud, M. Caminada, C. Cayrol, S. Doutre, M. Lagasquie, H. Prakken, and G. Vreeswijk. Towards a consensual formal model: inference part. Technical report, Deliverable D2.2: Draft Formal Semantics for Inference and Decision-Making. ASPIC project, 2004.

[3] L. Amgoud and C. Cayrol. Inferring from inconsistency in preference-based argumentation frameworks. *International Journal of Automated Reasoning*, Volume 29, N2:125–169, 2002.

[4] L. Amgoud and H. Prade. Using arguments for making decisions. In *Proceedings of the 20th Conference on Uncertainty in Artificial Intelligence*, pages 10–17, 2004.

[5] C. Boutilier. Towards a logic for qualitative decision theory. In *Proceedings of the 4th International Conference on Knowledge Representation and Reasoning*, pages 75–86, 1994.

[6] D. Dubois, L. Godo, H. Prade, and A. Zapico. On the possibilistic decision model: from decision under uncertainty to case-based decision. *Int. J. of Uncert., Fuzziness and Knowledge-Based Syst.*, 7:6:631–670, 1999.

[7] D. Dubois, H. Prade, and R. Sabbadin. *Decision-theoretic foundations of qualitative possibility theory*, volume 128. European Journal of Operational Research, 2001.

[8] P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77:321–357, 1995.

[9] H. Fargier and R. Sabbadin. Qualitative decision under uncertainty: back to expected utility. In *Proceedings of the International Joint Conference on Artificial Intelligence, IJCAI'03*, 2003.

[10] H. Prakken and G. Sartor. Argument-based extended logic programming with defeasible priorties. *Journal of Applied Non-Classical Logics*, 7:25–75, 1997.