Argument Theory Change Through Defeater Activation

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Abstract

Argument Theory Change applies classic belief change concepts to the area of argumentation. This intersection of fields takes advantage of the definition of a Dynamic Abstract Argumentation Framework, in which an argument is either active or inactive, and only in the former case it is taken into consideration in the reasoning process. An approach for an argument revision operator defined through deactivation of arguments can be found in the literature. The present article is inspired by that approach, and complements it by considering activation of arguments to define a revision operator.

1 Introduction and Background

This article presents a new approach to Argument Theory Change (ATC) [Moguillansky *et al.*, 2008; Rotstein *et al.*, 2008b], where belief change concepts [Alchourrón *et al.*, 1985] are translated to the field of argumentation. Here we will use the dynamic abstract argumentation framework given in [Rotstein *et al.*, 2008b], which extends Dung's framework [Dung, 1995] in order to consider (1) subarguments (internal, necessary parts of an argument that are arguments by themselves), and (2) a set of active arguments (those enabled to be used in the reasoning process). So far, the main contribution provided by ATC is a revision operator at argument level that revises a theory by an argument seeking its warrant. In the aforementioned articles, in order to achieve warrant, a revision operator was defined through deactivation of arguments. This means that some arguments are not taken into account by the reasoning process in order to ensure warrant of the argument at issue. In the present article, we propose a complementary approach, where the revision involves only activation of arguments. We claim that this approach complements the one based on deactivation because sometimes it is either not feasible to deactivate arguments or the activation of another provokes less change. Hence, the present approach completes the path towards a hybrid approach for ATC.

The translation of concepts from the deactivating approach brings about several differences that have to be taken care of to preserve the spirit of the revision operator. Although a strong analogy is sought and every notion finally finds its counterpart, it is necessary to establish different conditions in order to guarantee a successful revision. It will be clear that the arguments to be activated have to be defeaters of those interfering with the warrant of the argument at issue.

It is important to notice that the concepts here discussed increase the difficulty when compared to those in the complementary deactivating approach. There, incisions over arguments result in a theory T' that is "smaller" than the original one T; that is, every active argument in T' is also present in T. In opposition to this, the activating approach here presented increases the amount of active arguments to achieve warrant. However, since defeaters might be unavailable to be activated, the revision could be not always successful. Moreover, throughout the article it will be clear that each activation of an argument results in the possible activation of more arguments, thus provoking preexistent arguments to play additional roles.

Once again, the need to formalize the activating approach comes from two standpoints: it allows to cope with the fact that deactivation of arguments may not precisely reflect the evolving situation in the modeled world, and it also is mandatory to finally achieve a hybrid approach to handle evolution. Such an approach would combine activations and deactivations to perform the revision. The reader should be aware that this article does not pursue a complete formalization according to the classical theory of belief revision. Thus, no representation theorems nor characterization postulates are to be defined here. Instead, we look at the process of change from the argumentation side, following a practical take on this matter. However, the usual principles of change from belief revision were considered to specify this theory, as is the case of minimal change. The full formalization of the operators of change here proposed should conform to a set of readapted (in the sense of the argumentation theory) postulates by following the basic principles of change. Finally, the corresponding representation theorems would be formalized from such postulates. This work is underway in this line of research.

2 A Dynamic Abstract Framework

In this article, *arguments* are used in the usual sense, interpreted as a reason for a certain claim from a set of

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premises. In abstract argumentation [Chesñevar *et al.*, 2000; Prakken and Vreeswijk, 2000] these features are abstracted away; however, we will include them into the representation formalism. Moreover, we will be interested in the consideration of *subarguments*, which are the portions of an argument that compose an argument by themselves. Arguments with no subarguments are going to be called *atomic arguments*. From now on, we will say "arguments" to refer both to atomic and non-atomic arguments.

A single atomic argument $\mathcal{B}$ with unsatisfied premises could need other atomic arguments with the only purpose of satisfying its premises. These arguments (along with $\mathcal{B}$) are aggregated to conform a *non-atomic argument* $\mathcal{B}'$ standing for the same claim of $\mathcal{B}$ but containing no unsatisfied premises. In such a case, $\mathcal{B}'$ will be referred as a *full argument*, *i.e.*, arguments without premises to be satisfied; otherwise, it is considered a *partial argument*. This classification is applicable to every argument, including atomic ones.

Definition 1 (Full/Partial Argument) *An argument $\mathcal{A}$ is a **full argument** iff $\mathcal{A}$ has no unsatisfied premises. If $\mathcal{A}$ has at least one unsatisfied premise, it is a **partial argument**.*

In this article we use an extension to Dung's framework that is able to cope with the dynamics of arguments through the consideration of a *set of active arguments*. Arguments inside this set represent the current state of the world and are the only ones to be considered to compute warrant. Recall that the universal set of arguments holds every conceivable argument that could be used by the inference machinery. Therefore, at a given moment, those arguments that are inactive represent reasons that, though valid, cannot be taken into consideration due to the current context. For instance, the presentation of an appeal outside the acceptable period of time would render the argument for the appeal inactive. That is, although the appeal constitutes a valid reason, it is incompatible with the current state of the world. In this particular example, inactiveness is established by a time condition; however, other conditions could be considered. This correlation between triggering conditions and in/activeness of arguments lies beyond the scope of this article. A similar concept to inactive arguments is given in [Wyner and Bench-Capon, 2008] to *inadmissible arguments*.

Activation and deactivation of arguments are thus assumed to be determined from an external mechanism. In some domains, an agent might have the capability of activating and/or deactivating arguments (*e.g.*, argumentation-based dialogues), and therefore the challenge is to decide what kind of change has to be performed, *i.e.*, what to de/activate. This is the point in which ATC enters the scene, allowing to handle activation and deactivation of arguments in a proper manner, seeking for a concrete objective. (More on this can be found in the worked example in Sec. 4.) Another scenario in which ATC would perform well is hypothetical reasoning: once a goal is set (in terms of the warrant of an argument), ATC would determine the necessary changes for the argument at issue to be warranted, stating the subgoals that ensure the achievement of the final goal. These subgoals will translate into de/activation of arguments. The extension to Dung's framework introduced next is similar to the one used in [Rotstein *et al.*, 2008b] and is a simplified version of the framework presented in [Rotstein *et al.*, 2008a].

Definition 2 (DAF) *A **dynamic abstract argumentation framework** (DAF) is a tuple $\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle[\mathbb{A}]$, where $\mathbb{U}$ is a finite set of arguments called **universal**, $\mathbb{A} \subseteq \mathbb{U}$ is called the **set of active arguments**, $\hookrightarrow$ denotes the **attack relation** over $\mathbb{U}$, and $\sqsubseteq$ denotes the **subargument relation** over $\mathbb{U}$.*

As said before, a non-atomic argument $\mathcal{B}$ is composed by other (atomic) arguments to support $\mathcal{B}$ premises, therefore, it is always the case that there exists a $\mathcal{B}' \sqsubseteq \mathcal{B}$ such that the claims of both $\mathcal{B}'$ and $\mathcal{B}$ coincide. For instance, consider an argument $\mathcal{B}$: "*the fact that Steven did not turn in the report indicates that he is irresponsible; since Steven is irresponsible, he should be fired*". There we can identify two subarguments $\mathcal{B}_1$: "*failing in turning in reports suggests irresponsibility*" and $\mathcal{B}_2$: "*irresponsible people should be fired*". Note that $\mathcal{B}_2$'s claim –to fire the person– coincides with $\mathcal{B}$'s claim.

Hence, it is clear that every non-atomic argument $\mathcal{B}$ has an associated atomic providing the claim. This atomic argument may be identified through a function *top argument* $\mathsf{top} : \mathbb{U} \rightarrow \mathbb{U}$ such that $\mathsf{top}(\mathcal{B}) = \mathcal{B}'$ iff $\mathcal{B}$ and $\mathcal{B}'$s claims are equal. Note that the top argument is the only atomic argument not supporting another atomic argument within the argument it is a subargument of. In addition, the *set of atomic subarguments* composing a given argument $\mathcal{A}$ can be obtained through the function $\mu : \mathbb{U} \rightarrow \mathcal{P}(\mathbb{U})$ such that $\mu(\mathcal{A}) = \{\mathcal{A}' | \mathcal{A}'$ is atomic and $\mathcal{A}' \sqsubseteq \mathcal{A}\}$. Note that, in general, $\mathcal{A} \notin \mu(\mathcal{A})$ except for the case $\mu(\mathcal{A}) = \{\mathcal{A}\}$ when $\mathcal{A}$ is atomic. Finally, it is worthy to mention that the notion of support among subarguments within an argument is similar to that of the bipolar argumentation frameworks [Amgoud *et al.*, 2008]; however an important difference is that we consider subarguments as an essential part of the argument they belong to and, as such, the lack of a subargument would suppress the whole argument.

It can be shown that the subargument relation $\mathcal{A} \sqsubseteq \mathcal{B}$ may be interpreted as $\mu(\mathcal{A}) \subseteq \mu(\mathcal{B})$. Besides, in this work we will assume that the attack relation $\hookrightarrow$ is defined over atomic arguments, and expanded to the non-atomic arguments containing them. This principle, named *conflict inheritance* [Martínez *et al.*, 2007], characterizes the behavior of the framework regarding conflict management, and is used to identify to which part of the argument the attack is directed to.

(Conflict Inheritance) $\mathcal{A} \hookrightarrow \mathcal{B}$ iff $\exists \mathcal{B}' \in \mu(\mathcal{B})$ such that $top(\mathcal{A}) \hookrightarrow \mathcal{B}'$.

The dynamics of arguments are handled through the consideration of a set $\mathbb{A}$ of active arguments. Having both the universal set of arguments and the subset of the currently active ones allows us to identify the subset of *inactive arguments*, *i.e.*, $\mathbb{I} = \mathbb{U} \setminus \mathbb{A}$. The set of inactive arguments will contain the remainder of arguments (within the universal set) that is not considered by the argumentative process at a specific instant. Here, a second principle characterizing the activation of arguments is introduced as *activeness propagation*.

(Activeness Propagation) $\mathcal{B} \in \mathbb{A}$ iff $\mu(\mathcal{B}) \subseteq \mathbb{A}$.

A group of arguments being active determines that an argument containing them and only them is going to be active.

Furthermore, a non-atomic argument becoming active makes all of its subarguments to become active. This also works the other way around: if a subargument $\mathcal{A}$ of an argument $\mathcal{B}$ is set inactive, then every superargument of $\mathcal{A}$ is set inactive, including $\mathcal{B}$. Moreover, the inactiveness of $\mathcal{A}$ implies that at least one subargument of it is inactive.

Usually in the literature, the acceptability analysis over an argumentation framework is made over the graph of arguments implicit from the framework. In this article, we will focus on the warrant status of a single argument. Hence, we will build and evaluate a *dialectical tree* rooted in the argument under study in order to determine whether it can be found as warranted. As will be explained later, a tree is conformed by a set of *argumentation lines*. The following definitions are inspired from those in [Chesñevar and Simari, 2007].

We will call an *argumentation line* to a non-empty sequence of arguments λ from a DAF ϕ, where each argument in λ attacks its predecessor in the line. The first argument is called the *root*, and the last one, the *leaf* of λ. The domain of all argumentation lines in ϕ is noted as $\mathfrak{Lines}_\phi$. This set defines a domain onto which different constraints can be defined. As such constraints are related to sequences which resemble an argumentation dialogue between two parties, we call them *dialectical constraints*, and will be useful to determine whether an argumentation line is *acceptable*. Therefore, a dialectical constraint $\boldsymbol{c}$ in the context of ϕ is any function $\boldsymbol{c} : \mathfrak{Lines}_\phi \rightarrow \{true, false\}$. In what follows, we will assume a dialectical constraint that avoids the construction of circular argumentation lines, keeping them finite. Finally, the framework enriched with dialectical constraints will be referred as *argumentation theory*.

Definition 3 (DAT) *A **dynamic argumentation theory** (DAT) T is a pair $(\phi, \mathbf{DC})$, where ϕ is a DAF closed under activeness propagation and conflict inheritance, and $\mathbf{DC}$ is a finite set of dialectical constraints.*

An operator $\mathrm{C}_{\mathfrak{ap}} : \mathcal{P}(\mathbb{U}) \times \mathsf{T} \rightarrow \mathcal{P}(\mathbb{U})$ is assumed to implement the *closure under activeness propagation* required by Def. 3. Thus, given a DAT $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq\rangle[\mathbb{A}], \mathbf{DC})$, it follows $\mathbb{A} = \mathrm{C}_{\mathfrak{ap}}(\mathbb{A}, \mathsf{T})$. The attack relation is assumed to be analogously closed under conflict inheritance. To represent change over the set of active arguments we assume an *activation operator* $\boxplus^{\mathsf{T}} : \mathcal{P}(\mathbb{U}) \times \mathcal{P}(\mathbb{U}) \rightarrow \mathcal{P}(\mathbb{U})$ such that $\mathbb{A} \boxplus^{\mathsf{T}} \mathsf{Args} = \mathrm{C}_{\mathfrak{ap}}(\mathbb{A} \cup \mathsf{Args}, \mathsf{T})$, with $\mathsf{Args} \subseteq \mathbb{U}$.

Definition 4 (Acceptable Argumentation Line) *Given a DAT T, an argumentation line λ is **acceptable** wrt. T* iff *every argument in λ is full and $\boldsymbol{c}(\lambda) = true$ for every $\boldsymbol{c} \in \mathbf{DC}$. The domain of all acceptable argumentation lines in T is noted as $\mathfrak{Lines}^{\mathbb{U}}_{\mathsf{T}}$. The domain $\mathfrak{Lines}^{\mathbb{A}}_{\mathsf{T}} \subseteq \mathfrak{Lines}^{\mathbb{U}}_{\mathsf{T}}$ will enclose every acceptable line containing only active arguments.*

From now on, given a theory T, to refer to an argument $\mathcal{A}$ belonging to a line $\lambda \in \mathfrak{Lines}^{\mathbb{A}}_{\mathsf{T}}$, we will overload the membership symbol and write "$\mathcal{A} \in \lambda$", and will refer to λ simply as argumentation line assuming it is acceptable. Finally, the function $\mathsf{lineArgs} : \mathfrak{Lines}^{\mathbb{A}}_{\mathsf{T}} \rightarrow \mathcal{P}(\mathbb{U})$ returns the set of arguments belonging to a line.

Definition 5 (Upper Segment) *Given a DAT T and an acceptable argumentation line $\lambda \in \mathfrak{Lines}^{\mathbb{A}}_{\mathsf{T}}$ such that $\lambda = [\mathcal{B}_1, \ldots, \mathcal{B}_n]$, the **upper segment** of λ wrt. $\mathcal{B}_i$ $(1 \leq i \leq n)$ is defined as $\lambda^{\uparrow}[\mathcal{B}_i] = [\mathcal{B}_1, \ldots, \mathcal{B}_i]$. The **proper upper segment** of λ wrt. $\mathcal{B}_i$ is defined as $\lambda^{\uparrow}(\mathcal{B}_i) = [\mathcal{B}_1, \ldots, \mathcal{B}_{i-1}]$.*

In the sequel, we will refer to both proper and non-proper upper segments simply as upper segment and will be distinguishable only through its notation. Acceptable argumentation lines rooted in a common argument will be included into *bundle sets*. This notion will allow the formalization of dialectical trees, which in turn will be the source of analysis for any adopted marking criterion.

Definition 6 (Bundle Set) *Given a DAT T, a set $\mathcal{S}_{\mathsf{T}}(\mathcal{A}) = \{\lambda_1, \ldots, \lambda_n\}$ of all the acceptable argumentation lines from $\mathfrak{Lines}^{\mathbb{A}}_{\mathsf{T}}$ rooted in a given argument $\mathcal{A}$ is called a **bundle set** for $\mathcal{A}$ from T* iff *each $\lambda_i \in \mathcal{S}_{\mathsf{T}}(\mathcal{A})$ is **exhaustive**: there is no $\lambda_j \in \mathcal{S}_{\mathsf{T}}(\mathcal{A})$ such that $\lambda^{\uparrow}_i(\mathcal{B}) = \lambda_j$, for some $\mathcal{B} \in \lambda_i$ $(1 \leq i, j \leq n)$.*

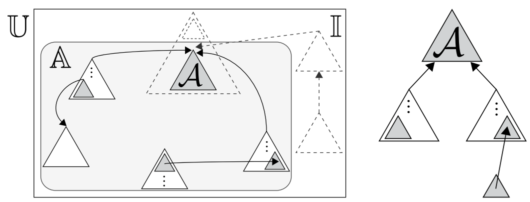
Therefore, since $\mathcal{S}_{\mathsf{T}}(\mathcal{A})$ contains all exhaustive lines (***i.e.***, no more arguments can be added to them) rooted in $\mathcal{A}$ from T, $\mathcal{S}_{\mathsf{T}}(\mathcal{A})$ will be used to build a *dialectical tree* rooted in $\mathcal{A}$. The notion of dialectical tree follows the usual intuitions that can be found on the literature [Prakken and Vreeswijk, 2000] reified to our formalism.

Definition 7 (Dialectical Tree) *Given a DAT $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq\rangle[\mathbb{A}], \mathbf{DC})$, a **dialectical tree** $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$ rooted in $\mathcal{A}$ is determined by a bundle set $\mathcal{S}_{\mathsf{T}}(\mathcal{A})$. For an **inner node** $\mathcal{B} \in \lambda$, $\lambda \in \mathcal{S}_{\mathsf{T}}(\mathcal{A})$, a **child** of $\mathcal{B}$ is every argument $\mathcal{D} \in \lambda'$, $\lambda' \in \mathcal{S}_{\mathsf{T}}(\mathcal{A})$ such that $\mathcal{D} \hookrightarrow \mathcal{B}$ and $\lambda'^{\uparrow}(\mathcal{D}) = \lambda^{\uparrow}[\mathcal{B}]$. The **leaves** in $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$ are the leaves of each argumentation line in $\mathcal{S}_{\mathsf{T}}(\mathcal{A})$. The domain of all dialectical trees in T will be noted as $\mathfrak{Tree}_{\mathsf{T}}$.*

We will overload the membership symbol and write "$\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$" when the line λ belongs to the bundle set associated to the tree $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$. Dialectical trees allow to determine whether the root node of the tree is warranted or not. This evaluation will weigh all the information present in the tree on behalf of a *marking criterion* to evaluate each argument in the tree, in particular the root. We will abstract away from the marking criterion, relying on a function $\mathsf{Mark} : \mathfrak{Tree}_{\mathsf{T}} \rightarrow \{D, U\}$ returning the mark of the root, where U (resp., D) denotes an undefeated (resp., defeated) argument.

Definition 8 (Warrant) *Let T be a DAT and* Mark*, a marking criterion for T. A full active argument $\mathcal{A}$ is **warranted** wrt.* Mark *in T* iff *the dialectical tree $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$ is such that $\mathsf{Mark}(\mathcal{T}_{\mathsf{T}}(\mathcal{A})) = U$. Whenever $\mathsf{Mark}(\mathcal{T}_{\mathsf{T}}(\mathcal{A})) = U$, the tree $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$ will be called a **warranting tree**; otherwise, it will be called a **non-warranting tree**.*

Example 1 *The digraph depicted in Figure 1(a) describes a DAF, where triangles are arguments and arcs denote attack. The set $\mathbb{A}$ of active arguments is shown, along with the universal $\mathbb{U}$, and the set $\mathbb{I}$ of inactive arguments (dashed triangles). Subarguments were drawn inside their superarguments. Note that $\mathcal{A}$'s superargument is inactive because it contains an inactive subargument. Figure 1(b) shows a dialectical tree spanning the graph from argument $\mathcal{A}$. Observe that attacks between inactive and active arguments, but inactive arguments are not considered in the tree. Consider a* *marking function where each node of the tree is undefeated* iff *either it is a leaf or all of its defeaters are defeated; hence, the tree is non-warranting. This status changes if we activate, for instance, a defeater for the root's left defeater.*



(a) (b)

Figure 1: (a) DAT example (b) Tree spanning from $\mathcal{A}$

3 Argument Theory Change

We have a theory T to be revised by an argument $\mathcal{A}$ in such a way that $\mathcal{A}$ ends up warranted from a resulting theory T_R. The first step in the revision process is to add $\mathcal{A}$ to T, thus obtaining a (temporary) theory T'. This theory is deemed as temporary, since $\mathcal{A}$ might probably not be a warrant from T', and usually $\mathsf{T}' \neq \mathsf{T}_R$. Then, a *temporary dialectical tree* $\mathcal{T}_{\mathsf{T}'}(\mathcal{A})$ rooted in $\mathcal{A}$ is built from T'. Next, we have to identify those argumentation lines from $\mathcal{T}_{\mathsf{T}'}(\mathcal{A})$ that mark the root argument as defeated. These lines are grouped in the set $Att_{\mathsf{T}}(\mathcal{A})$ of *attacking lines* and are *altered* by the change machinery. The alteration aims to make an argumentation line to cease being an attacking line. For that purpose we will rely on the addition of a defeater.

Sometimes attacking lines are not independent wrt. the remaining lines in the tree. Therefore, an alteration could bring about further difficulty, and the correctness of the change process would have to be ensured by principles avoiding these side-effects. Nonetheless, some alterations could collaterally provoke other desirable alterations. The former side-effects must be avoided, whereas the latter could be pursued. When no side-effects occur, turning an attacking line into non-attacking is simple. As will be clear next, this issue could be solved by adding a defeater to a *con* argument. However, having no side-effects is not frequent, since arguments could share subarguments. How to overcome a bad side-effect will be clear by the end of this section.

3.1 Preliminaries

When activating arguments, often the theory will have to make a choice among several options. For this matter, we will rely on a criterion that helps in making this decision.

Definition 9 (Argument Activation Criterion) *Given a DAT* $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle[\mathbb{A}], \mathbf{DC})$, *and two arguments* $\{\mathcal{A}, \mathcal{B}\} \subseteq \mathbb{I}$, *the* ***argument activation criterion*** *"*$\prec_{\mathsf{T}}$*" (towards minimal change) defines a total order over* $\mathbb{U}$, *such that* $\mathcal{A} \prec_{\mathsf{T}} \mathcal{B}$ iff *activating* $\mathcal{A}$ *provokes less change than activating* $\mathcal{B}$.

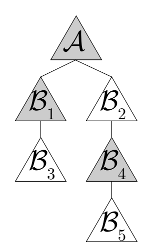
In this article, we will abstract away from this criterion. For instance, if we were stating that an argument's amount of change is directly related to the number of lines a theory would have when adding it, the definition would be: $\mathcal{A} \prec_{\mathsf{T}} \mathcal{B}$ iff $|\mathfrak{Lines}^{\mathbb{A}'}_{\mathsf{T}_{\mathcal{A}}}| \leq |\mathfrak{Lines}^{\mathbb{A}''}_{\mathsf{T}_{\mathcal{B}}}|$, where $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle[\mathbb{A}], \mathbf{DC})$, $\mathcal{A}, \mathcal{B} \in \mathbb{I}$, $\mathsf{T}_{\mathcal{A}} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle[\mathbb{A}'], \mathbf{DC})$, $\mathsf{T}_{\mathcal{B}} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle[\mathbb{A}''], \mathbf{DC})$, $\mathbb{A}' = \mathbb{A} +^{\mathsf{T}} \{\mathcal{A}\}$, and $\mathbb{A}'' = \mathbb{A} +^{\mathsf{T}} \{\mathcal{B}\}$. Next, we introduce the notion of *attacking line*. Intuitively, this kind of argumentation lines are the ones *responsible for a non-warranting dialectical tree*. This set comprehends all the lines in a tree such that, without them, the tree would be warranting.

Definition 10 (Set of Attacking Lines) *Given a DAT* $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle[\mathbb{A}], \mathbf{DC})$, *a dialectical tree based on a bundle set* $\mathcal{S}_{\mathsf{T}}(\mathcal{A})$, *and an argument activation criterion* $\prec_{\mathsf{T}}$, *the* ***set of attacking lines*** $Att_{\mathsf{T}}(\mathcal{A}) \subseteq \mathcal{S}_{\mathsf{T}}(\mathcal{A})$ *is the minimal subset wrt. set inclusion determined by* $\prec_{\mathsf{T}}$ *such that either* $Att_{\mathsf{T}}(\mathcal{A}) = \mathcal{S}_{\mathsf{T}}(\mathcal{A})$ *or the tree built from the bundle set* $\mathcal{S}_{\mathsf{T}}(\mathcal{A}) \setminus Att_{\mathsf{T}}(\mathcal{A})$ *warrants* $\mathcal{A}$.

By the end of the definition we have two separate cases: the first one isolates a scenario in which every line in the tree is interfering with $\mathcal{A}$'s warrant and therefore $\mathcal{S}_{\mathsf{T}}(\mathcal{A}) \setminus Att_{\mathsf{T}}(\mathcal{A})$ would be an empty set –recall that the set $Att_{\mathsf{T}}(\mathcal{A})$ identifies those lines preventing $\mathcal{A}$ to be warranted; the second case checks if the remaining lines would warrant $\mathcal{A}$. The objective of this definition is to identify attacking lines. Note that the set $\mathcal{S}_{\mathsf{T}}(\mathcal{A})$ (which is a subset of $Att_{\mathsf{T}}(\mathcal{A})$) could be non-maximal, and thus since it would not conform a bundle set, no dialectical tree could be associated to it. That is, the removal of the attacking lines from the bundle set of a non-warranting tree is not intended to conform a proper change operation. The following example will be worked throughout the rest of the article to illustrate definitions. However, for the sake of simplicity, we will consider subarguments only when this contributes to the comprehension of the theory. Subarguments will be included on a worked example in Sec. 4.

Example 2

Assuming the marking given in Example 1, consider a DAT T *yielding the tree on the right, where gray (resp., white) triangles denote defeated (resp., undefeated) arguments. There is one attacking line, therefore* $Att_{\mathsf{T}}(\mathcal{A}) = \{[\mathcal{A}, \mathcal{B}_2, \mathcal{B}_4, \mathcal{B}_5]\}$. *If we activate a defeater for* $\mathcal{B}_2$ *in* T, *we would generate a new line within a new tree having no attacking lines. If we instead add a defeater for* $\mathcal{B}_5$, *the line that was attacking is "extended" and again, in the resulting tree, there would be no attacking lines. On the other hand, if we activate a defeater for* $\mathcal{B}_4$, *we would generate another attacking line.*



As shown by this example, the activation of defeaters has to be performed carefully, looking for a tree with no attacking lines. Moreover, the activation of a defeater might trigger the activation of more arguments, making things more difficult. This will be addressed later in the article. Regarding the recognition of attacking lines by considering the tree resulting from the removal from Def. 10, it is important to stress that this is a hypothetical scenario that is useful to isolate the causes for a non-warranting tree. The following lemma formalizes this notion.

Lemma 1 *A dialectical tree $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$ warrants argument $\mathcal{A}$* iff $Att_{\mathsf{T}}(\mathcal{A}) = \emptyset$.

Proof sketch: *By Def. 10, whenever $Att_{\mathsf{T}}(\mathcal{A}) = \emptyset$ it holds that the tree built from $\mathcal{S}_{\mathsf{T}}(\mathcal{A}) \setminus Att_{\mathsf{T}}(\mathcal{A})$ warrants $\mathcal{A}$,* i.e., *the original tree does, since $Att_{\mathsf{T}}(\mathcal{A})$ is required to be minimal. Hence, a warranting tree and a tree with an empty set $Att_{\mathsf{T}}(\mathcal{A})$ are equivalent notions.*

3.2 A Change Method Using Activation of Arguments

As already mentioned, the objective of this article is to define a change operator that revises a DAT by an argument, making the necessary modifications to the theory to warrant that argument. The core of the change machinery involves alteration of attacking lines. In [Rotstein *et al.*, 2008b], ATC was introduced along with a deactivating approach for revising an argumentation system by an argument. In this work, by alteration we refer to the addition of an argument (*i.e.*, a defeater) into a line. Alteration of lines comes from changes applied to the set of active arguments in the theory; that is, arguments cannot be simply added to the tree. Hence, since an argument could appear in different positions in several lines in a tree, an alteration of a line could result in collateral alterations of other lines. Their treatment is included later in this article.

The addition of a defeater $\mathcal{D}$ for an argument in a line λ results in an extension: if $\mathcal{D}$ is attacking the leaf of λ, the whole line ends up extended, but if $\mathcal{D}$ attacks an argument placed strictly above that leaf, a new argumentation line arises by extending an upper segment of λ. It is important to note that the activation of $\mathcal{D}$ not only attaches $\mathcal{D}$ to λ, but it also includes the addition of $\mathcal{D}$'s (active) defeaters, and these defeaters bring their (active) defeaters, and so on, and finally an entire subtree rooted in $\mathcal{D}$ sprouts from the activation of this argument. This subtree could contain arguments that were already active, as well as arguments that ended up activated by virtue of activeness propagation. Finally, when a defeater is to be activated to attack an active argument in a line λ, more arguments could be activated, some of which could also attack an argument in λ. The following function returns the set of inactive defeaters for a given argument $\mathcal{B}$.

Definition 11 (Set of Inactive Defeaters) *Let $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle [\mathbb{A}], \mathbf{DC})$ be a DAT, $\lambda \in \mathfrak{Lines}_{\mathsf{T}}^{\mathbb{A}}$, an argumentation line, and $\mathcal{B} \in \lambda$, an argument. The* ***set of inactive defeaters*** *is determined by the function* $\mathsf{idefs} : \mathbb{A} \times \mathfrak{Lines}_{\mathsf{T}}^{\mathbb{A}} \times \mathsf{T} \rightarrow \mathcal{P}(\mathbb{I})$ *is defined as:*

$$\mathsf{idefs}(\mathcal{B}, \lambda, \mathsf{T}) = \{\mathcal{D} \in \mathbb{I} \mid \mathcal{D} \text{ is full, } \mathcal{D} \hookrightarrow \mathcal{B}\}$$

If it is the case that $\mathcal{B} \notin \lambda$ then the function's outcome is set to $\mathsf{idefs}(\mathcal{B}, \lambda, \mathsf{T}) = \emptyset$.

In the deactivating approach to ATC, an attacking line ends up truncated due to the deactivation of a *con* argument. Here, the alteration of a given attacking line determines the alteration by activating a defeater for a *con* argument in the line. However, it is clear that this might be unachievable, such a defeater might not exist. As will be clear later, such exceptional situations keep the revision operation from succeeding.

Recall that the purpose of the selection of an argument in the deactivating approach is to identify, from that specific attacking line, the *con* argument to be deactivated. In the case of the activating approach here presented this intention differs in that the selected argument would end up having a new active defeater. In this sense, the appropriate selection of a *con* argument is determined as follows.

Definition 12 (Argument Selection Function) *Given a dialectical tree $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$ from a DAT $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle [\mathbb{A}], \mathbf{DC})$, and an argumentation line $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$, the* ***argument selection function*** *$\gamma : \mathfrak{Lines}_{\mathsf{T}}^{\mathbb{A}} \rightarrow \mathbb{A}$ is guided by the criterion "$\prec_{\mathsf{T}}$" and defined as:*

$$\gamma(\lambda) = \begin{cases} \mathcal{B} \in \lambda^{-} & \text{if } \mathsf{idefs}(\mathcal{B}, \lambda, \mathsf{T}) \neq \emptyset \\ \mathsf{head}(\lambda) & \text{otherwise.} \end{cases}$$

The criterion determines whether the activation of an argument implies less change than the activation of another one. Thus, when comparing two active arguments such as in the case of the selection function, their sets of inactive defeaters have to be used. For instance, the criterion could evaluate which defeater provokes the least change, leading to the selection of the argument it defeats. Finally, note that the selection will map to the line's head only when there will be no full inactive defeaters for any *con* argument in the line. The following function checks whether the alteration of an argumentation line brings new attacking lines.

Definition 13 (Attack-Free Function)
Let $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle [\mathbb{A}], \mathbf{DC})$ be a DAT, $\lambda \in \mathfrak{Lines}_{\mathsf{T}}^{\mathbb{A}}$, an argumentation line, and $\mathcal{B} \in \mathbb{A}$ and $\mathcal{D} \in \mathbb{I}$, two arguments. The ***attack-free function*** $\mathsf{att\_free} : \mathbb{I} \times \mathbb{A} \times \mathfrak{Lines}_{\mathsf{T}}^{\mathbb{A}} \times \mathsf{T} \rightarrow \{true, false\}$ *is defined as:*

$$\mathsf{att\_free}(\mathcal{D}, \mathcal{B}, \lambda, \mathsf{T}) = \begin{cases} true & \text{if } S \cap Att_{\mathsf{T}'}(\mathcal{A}) = \emptyset \text{, such} \\ & \text{that } \mathcal{B} \in \lambda \text{ and} \\ & \mathcal{D} \in \mathsf{idefs}(\mathcal{B}, \lambda, \mathsf{T}) \\ false & \text{otherwise,} \end{cases}$$

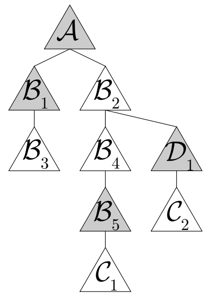
where $\mathsf{T}' = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle [\mathbb{A}']$, with $\mathbb{A}' = \mathbb{A} +^{\mathsf{T}} \{\mathcal{D}\}$, $\mathcal{A} = \mathsf{head}(\lambda)$, and:

$$S = \{\lambda' \in \mathfrak{Lines}_{\mathsf{T}'}^{\mathbb{A}'} \mid \lambda' \text{ is exhaustive in } \mathfrak{Lines}_{\mathsf{T}'}^{\mathbb{A}'} \text{ and } \lambda'^{\uparrow}[\mathcal{B}] = \lambda^{\uparrow}[\mathcal{B}]\}$$

When altering a line, other alterations could collaterally appear (see Def. 15). It is required then to analyze all the altered fragments in a single line λ resulting from an alteration in the same line λ. This situation ends up in multiple generation of new argumentation lines, some of them extending the original λ. Hence, through the attack-free function it is analyzed whether each new line extending λ could threat the warrant status of the root. That means that attack-free will be true only when the defeater $\mathcal{D}$ is a "good" defeater of $\mathcal{B}$ wrt. minimal change. This is exemplified next.

Example 3 (Continues from Ex. 2)

Given the DAT T from Ex. 2, since we have the attacking line $\lambda = [\mathcal{A}, \mathcal{B}_2, \mathcal{B}_4, \mathcal{B}_5]$, then in order to change the mark *of $\mathcal{A}$, we have to select an argument there to perform an alteration. Let assume that the criterion indicates that $\mathcal{B}_2$ is the most suitable argument for selection (*i.e., *$\mathcal{B}_2 \prec_{\mathsf{T}} \mathcal{B}_5$). Thus, let consider* $\mathsf{idefs}(\mathcal{B}_2, \lambda, \mathsf{T}) = \{\mathcal{D}_1, \mathcal{D}_2\}$ *with $\mathcal{D}_1 \prec_{\mathsf{T}} \mathcal{D}_2$, where the activation of $\mathcal{D}_1$ would determine a DAT T' and $\{\mathcal{C}_1, \mathcal{C}_2\} \subset \mathbb{A}{+}^{\mathsf{T}}\{\mathcal{D}_1\} = \mathbb{A}'$. The collateral activation of $\mathcal{C}_1$ could appear if we consider $\mathcal{D}_1 \sqsubseteq \mathcal{C}_1$, being the reimaining part of $\mathcal{C}_1$ already active; then, $\mathcal{C}_1$ is activated as a consequence of the activation of $\mathcal{D}_1$. Regarding $\mathcal{C}_2$, it could be already active, but since it is assumed that it only defeats $\mathcal{D}_1$, the activation of $\mathcal{D}_1$ would provoke $\mathcal{C}_2$ to be included in the resulting tree. Additionally, suppose that $\mathcal{C}_1 \hookrightarrow \mathcal{B}_5$, yielding the tree depicted on the right. From Def. 13 we have that $S = \{[\mathcal{A}, \mathcal{B}_2, \mathcal{B}_4, \mathcal{B}_5, \mathcal{C}_1], [\mathcal{A}, \mathcal{B}_2, \mathcal{D}_1, \mathcal{C}_2]\}$ and $Att_{\mathsf{T}'}(\mathcal{A}) = \{[\mathcal{A}, \mathcal{B}_2, \mathcal{D}_1, \mathcal{C}_2]\}$, thus* $\mathsf{att\_free}(\mathcal{D}_1, \mathcal{B}_2, \lambda, \mathsf{T}) = false$ *since $S \cap Att_{\mathsf{T}'}(\mathcal{A}) \neq \emptyset$. Finally, the mark of $\mathcal{A}$ remains unchanged. This example will be reconsidered later to achieve a successful revision.*



Given the selected argument, a set of inactive defeaters of it may be recognized in the framework and one of them appropriately chosen –through the argument activation criterion– to be activated. This will be referred as "padding" because we will be finding the missing parts of that argument in order to activate it. Consequently, by activeness propagation, the whole defeater will end up activated. Thereafter, this activation would end up in a possibly multiple activation of atomic arguments. The padding will be determined through an *argument padding function* which will identify the best defeater to be activated. As said before, it is possible to have a situation in which no defeater appears in the framework. In such a case the activating revision would not succeed (see Def. 16) given that the attacking line at issue will not be modified. In such a case the padding function will be empty, determining no argument to activate.

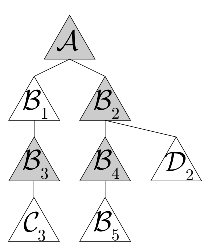
Definition 14 (Argument Padding Function) *Given a DAT $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle[\mathbb{A}], \mathbf{DC})$, a tree $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$, and an argument $\mathcal{B} \in \lambda$ selected through "γ" from the line $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$; the* ***argument padding function*** *$\sigma : \mathbb{A} \to \mathcal{P}(\mathbb{I})$ is defined as:*

$$\sigma(\mathcal{B}) = \begin{cases} \mu(\mathcal{D}) \setminus \mathbb{A} & \text{where } \mathcal{B} \in \lambda^- \text{ and } \mathcal{D} \in \mathsf{idefs}(\mathcal{B}, \lambda, \mathsf{T}) \\ & \text{ such that } \mathsf{att\_free}(\mathcal{B}, \mathcal{D}, \lambda, \mathsf{T}) = true, \\ \emptyset & \text{otherwise.} \end{cases}$$

The selection of the appropriate $\mathcal{D} \in \mathsf{idefs}(\mathcal{B}, \lambda, \mathsf{T})$ is led by "$\prec_{\mathsf{T}}$".

Example 4 (Continues from Ex. 3)

From $\gamma(\lambda) = \mathcal{B}_2$ we have that introducing $\mathcal{D}_1$ as a defeater for $\mathcal{B}_2$ would not satisfy the $\mathsf{att\_free}$ *function (the first case of the padding function), thus the defeater $\mathcal{D}_2$ would be looked to be padded, yielding the tree on the right. Note that $\{\mathcal{C}_3\} \subset \mathbb{A}{+}^{\mathsf{T}}\{\mathcal{D}_2\}$ and $\mathcal{C}_3 \hookrightarrow \mathcal{B}_3$. Now* $\mathsf{att\_free}(\mathcal{D}_2, \mathcal{B}_2, \lambda, \mathsf{T}) = true$*; nonetheless, from this alteration an attacking line arises from the addition of $\mathcal{C}_3$ to $\lambda_1 = [\mathcal{A}, \mathcal{B}_1, \mathcal{B}_3]$. Note that this attacking line has to be treated separately, since it is independent from λ. Such a side-effect will be treated by a principle explained later.*



The padding function returns an empty set when either it does not find a defeater, or every defeater found leads to an alteration yielding some attacking line. This means that there is no plausible padding for the selection under consideration. Thus, a new selection in the same line could solve this situation. The following principle provides such a mechanism, provoking an interaction between paddings and selections.

(Effective Padding) $\sigma(\gamma(\lambda)) \neq \emptyset$, where $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$.

The padding $\sigma(\gamma(\lambda))$ identifies the atomic arguments to be activated in order to activate a defeater $\mathcal{D}$. It may be the case that another inactive argument $\mathcal{C}$ is activated as a consequence of the activation of $\mathcal{D}$. This may happen when the portion of $\mathcal{D}$ to be activated contains all the inactive subarguments of $\mathcal{C}$. This situation is illustrated in Ex. 2, where a portion of the inactive defeater $\mathcal{D}_1$ is assumed to be the inactive part of $\mathcal{C}_1$, thus activating $\mathcal{D}_1$ implies the collateral activation of $\mathcal{C}_1$. A similar situation occurs in Ex. 4 with the activation of $\mathcal{D}_2$ and the collateral activation of $\mathcal{C}_3$. This side-effect of a padding is referred as *collateral padding*. Moreover, if $\mathcal{C}$ appears in the resulting dialectical tree after its activation, it could be generating new argumentation lines, which should be considered accordingly. That is, the alteration of the original line might end up as attacking in the resulting tree. To address this shortcoming we present a set of principles after formalizing the notion of collaterally padded defeater. Such collateral effects are treated by the preservation principle, explained later.

Definition 15 (Set of Collaterally Padded Defeaters) *Let $\mathcal{T}_{\mathsf{T}}(\mathcal{A})$ be a dialectical tree from a DAT $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle[\mathbb{A}], \mathbf{DC})$, and $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$, an argumentation line. The* ***set of collaterally padded defeaters*** $\mathsf{cpd} : \mathfrak{Lines}^{\mathbb{A}}_{\mathsf{T}} \times \mathsf{T} \to \mathcal{P}(\mathbb{I} \times \mathbb{A} \times \mathfrak{Lines}^{\mathbb{A}}_{\mathsf{T}} \times \mathsf{T})$ *from the padding $\sigma(\gamma(\lambda))$ is defined as:*

$$\mathsf{cpd}(\lambda, \mathsf{T}) = \{\langle \mathcal{D}, \mathcal{C}, \lambda', \mathsf{T} \rangle \mid \mathcal{D} \in \mathsf{idefs}(\mathcal{C}, \lambda', \mathsf{T}) \text{ and } \mathcal{D} \in \mathbb{A}{+}^{\mathsf{T}}\sigma(\gamma(\lambda))\}, \text{ where } \mathcal{C} \neq \sigma(\gamma(\lambda)).$$

A given tuple $\langle \mathcal{D}, \mathcal{C}, \lambda', \mathsf{T} \rangle$ represents the ***collaterally padded defeater (c.p.d.)*** *$\mathcal{D}$ for an argument $\mathcal{C}$ in a line λ' within the theory T. Argument $\mathcal{D}$ will be collaterally activated by $\sigma(\gamma(\lambda))$ once the revision is performed.*

Note that, in Ex. 4, $\mathcal{C}_3$ is a collaterally padded defeater: $\langle \mathcal{C}_3, \mathcal{B}_3, \lambda_1, \mathsf{T} \rangle \in \mathsf{cpd}(\lambda, \mathsf{T})$. A desirable condition to activate defeaters could be to avoid generating any collateral padding. This could be addressed through the optional *cautiousness* principle guiding the argument selection function.

(Cautiousness) For every $\mathcal{D} \in \mathsf{idefs}(\gamma(\lambda), \lambda, \mathsf{T})$ and every $\mathcal{C} \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$ there is no $\mathcal{B} \in \mathsf{idefs}(\mathcal{C}, \lambda', \mathsf{T})$, with $\mathcal{D} \neq \mathcal{B}$, such that $\mathcal{B} \in \mathbb{A}{+}^{\mathsf{T}}\mathcal{D}$.

By cautiousness we seek for a selection $\gamma(\lambda)$ such that every full defeater $\mathcal{D}$ of it can be activated while ensuring that it causes no collateral padding over an inactive full defeater $\mathcal{B}$ of some argument $\mathcal{C}$ in the tree.

Proposition 1 *Given a DAT* T*, for every* $\lambda \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$*, if the selection* $\gamma(\lambda)$ *guarantees cautiousness then* $\mathsf{cpd}(\lambda, \mathsf{T}) = \emptyset$*.*
Proof sketch: A selection satisfying cautiousness implies that no other defeater for an argument in the tree –apart from the one padded because of the selection itself– is activated. Therefore, its set of collaterally padded defeaters is empty.

Since we are looking for the warrant of the root argument in the dialectical tree, any collateral padding activating a *pro* argument might be helpful. Moreover, this kind of collateral paddings could be pursued with the explicit intention of modifying several attacking lines at once. This would be a clear advantage from the standpoint of minimal change. In this sense, an optional principle is introduced, requiring every collateral padding to occur over an attacking line and to coincide with the selection there.

(Profitability) If $\langle \mathcal{B}, \mathcal{C}, \lambda, \mathsf{T} \rangle \in \mathsf{cpd}(\lambda', \mathsf{T})$ then $\lambda \in Att_{\mathsf{T}}(\mathcal{A})$ and $\gamma(\lambda) = \mathcal{C}$, where $\lambda' \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$.

Example 5 *From Ex. 4, we have that the collaterally padded defeater* $\mathcal{C}_3$ *does not alter an attacking line, therefore profitability is not verified. Since there is no more defeaters for* $\mathcal{B}_2$*, the padding function returns an empty set. If effective paddings were a requirement, another selection would have to be made in order to satisfy both principles. The only option would be to change the selection in* λ*. Thus, the appropriate selection would be* $\mathcal{B}_5$*, for which a defeater should be padded.*

However, it could be the case that profitability cannot be satisfied. Therefore, collateral paddings should be controlled in order to avoid bad side-effects. When there is a collaterally padded defeater, the alteration of lines generated by it might include new attacking lines. Since these lines are not trivially taken into account to be altered, new selections have to be assigned to those lines. Note that these selections have to be arguments belonging to the original tree, since there is no way to handle changes that have not been actually made to the theory. Hence, we introduce the *preservation* principle.

(Preservation) If $\langle \mathcal{B}, \mathcal{C}, \lambda, \mathsf{T} \rangle \in \mathsf{cpd}(\lambda', \mathsf{T})$ then $\gamma(\lambda) \in \lambda^{\uparrow}[\mathcal{C}]$, where $\mathcal{C} \neq \mathcal{A}$, and $\lambda' \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$.

It is important to stress the point regarding selections when the collateral padding resulted in a new attacking line. It is required to select in the upper segment of $\mathcal{C}$, since below this point there could arise an entire subtree (not belonging to the original tree) containing these new lines for which $\lambda^{\uparrow}[\mathcal{C}]$ is common. Therefore, an alteration there would solve all of these emerging attacking lines. Furthermore, if there is another collaterally padded defeater for an argument in $\lambda^{\uparrow}[\mathcal{C}]$, preservation will take over the situation again, reassigning the selection to some upper segment within $\lambda^{\uparrow}[\mathcal{C}]$.

Example 6 (Continues from Ex. 4)
The alteration of λ_1 *from the collaterally padded defeater* $\mathcal{C}_3$ *(*i.e., $\langle \mathcal{C}_3, \mathcal{B}_3, \lambda_1, \mathsf{T} \rangle \in \mathsf{cpd}(\lambda, \mathsf{T})$*) would yield a new attacking line (*i.e., $\mathsf{att\_free}(\mathcal{C}_3, \mathcal{B}_3, \lambda_1, \mathsf{T}) = false$*)) that should be treated accordingly (*i.e., $\gamma(\lambda_1) \in \lambda^{\uparrow}[\mathcal{B}_3]$*) in order to satisfy preservation.*

Lemma 2 *Cautiousness implies profitability, and profitability implies preservation.*

Proof sketch: The first implication is trivial, since cautiousness ensures no collateral paddings arise; therefore, the antecedent of the implication in the profitability principle is false, rendering the whole formula true. The second implication is a bit trickier: both profitability and preservation have a common antecedent, so it has to be shown that is never the case that the consequent of profitability is true while the consequent of preservation remains false. For the consequent of profitability to be true, λ *has to belong to* $Att_{\mathsf{T}}(\mathcal{A})$ *and the selection there has to be* $\mathcal{C}$*. The only condition to be satisfied in the consequent of preservation is that the selection has to belong to* $\lambda^{\uparrow}[\mathcal{C}]$*, which includes* $\mathcal{C}$ *and thus it cannot be false.*

Through Lemma 2 it is clear that preservation is the least restrictive principle needed to handle the collateral padding threat. However, this does not ensure that a padding verifying it will guarantee the success of the revision. That is, since the modification of attacking lines is performed through the activation of defeaters, it is not feasible to assume their existence. When no defeater is available to be activated, attacking lines cannot be altered, thus the padding function returns an empty set, and the activating method inevitably fails. Hence, there is a way to identify whether the change operation will work.

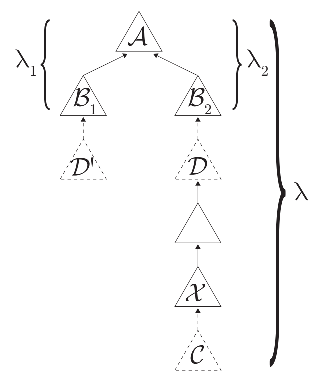


Figure 2: Schema for the non-cumulativity principle

In Schema 2, a new drawback to ATC is illustrated, in which collateral paddings occur extending at least twice the same argumentation line. This threat is called *cumulative collateral padding*. In the schema, from the selection $\gamma(\lambda_1) = \mathcal{B}_1$, argument $\mathcal{D}'$ is activated, provoking the collateral activation of $\mathcal{C}$. Besides, the selection $\gamma(\lambda_2) = \mathcal{B}_2$ provokes the activation of the defeater $\mathcal{D}$. Note that $\mathcal{C}$ would end up not only active, but also part of the resulting dialectical tree, since $\mathcal{D}$ is the link tying both parts of λ. This issue arises from different paddings, thus the only way to control it is from the union of all the arguments to be activated. Note that when the padding function over $\gamma(\lambda_2)$ chooses to activate $\mathcal{D}$, the att-free function is not aware of the collateral activation of $\mathcal{C}$, since it appears from an alteration in a different line. Therefore, a new principle called *non-cumulativity* is proposed in order to avoid the validation of selections and paddings leading to the described situation. Observe from the principle's formula that line λ is the inactive line in T that would end up completely activated if cumulative paddings were allowed. From the schema it is clear that, if such situation is not controlled through *non-cumulativity*, the resulting dialectical tree could end up non-warranting.

(Non-Cumulativity) If $\mathcal{D} \in \mathsf{idefs}(\gamma(\lambda_1), \lambda_1, \mathsf{T})$ and there is $\lambda \in \mathfrak{Lines}^{\mathbb{U}}_{\mathsf{T}}$ such that $\lambda = [\mathcal{B}_1, \ldots, \mathcal{B}_n, \mathcal{D}, \ldots, \mathcal{C}]$ then for every $\lambda_3 \in \mathcal{T}_{\mathsf{T}}(\mathcal{A})$ it holds that $\mathcal{D} \notin \mathbb{A} +^{\mathsf{T}} \sigma(\gamma(\lambda_3))$, where $\lambda_1 = [\mathcal{B}_1, \ldots, \mathcal{B}_n]$, $\mathcal{C} \in \mathbb{I}$, and $\mathcal{C} \in \mathbb{A} +^{\mathsf{T}} \sigma(\gamma(\lambda_2))$

Definition 16 (Warranting Padding Function) *An argument padding function "σ" is a* ***warranting padding function*** iff *it guarantees* ***effective padding****,* ***non-cumulativity*** *and either* ***preservation****,* ***profitability****, or* ***cautiousness****.*

Note that, by Lemma 2, the preservation principle is always required in order for a padding function to be warranting, since it is implied by the other two principles. The restrictions imposed by the warranting padding function ensure a successful revision, as will be clear later. Nonetheless, a padding function might not be warranting, thus a revision operation based on it is not always successful. In such a case, no change is performed over the original DAT. Next, we define the activating approach for the argument revision function, which requires the padding function to be warranting.

Definition 17 (AWR) *An* ***activating warrant-prioritized argument revision (AWR)*** *operator "$\times^{\omega}$" over $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsupseteq \rangle[\mathbb{A}], \mathbf{DC})$ by a full argument $\mathcal{A} \in \mathbb{U}$ is:*

$$\mathsf{T} \times^{\omega} \mathcal{A} = \begin{cases} (\langle \mathbb{U}, \hookrightarrow, \sqsupseteq \rangle[\mathbb{A}' +^{\mathsf{T}} \bigcup_{\lambda \in S} \sigma(\gamma(\lambda))], \mathbf{DC}) \\ \qquad \text{if "}\sigma\text{" is warranting} \\ \mathsf{T} \qquad \text{otherwise,} \end{cases}$$

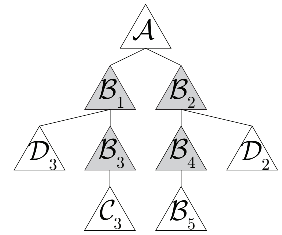
where $\mathbb{A}' = \mathbb{A} +^{\mathsf{T}} \{\mathcal{A}\}, \mathsf{T}' = (\langle \mathbb{U}, \hookrightarrow, \sqsupseteq \rangle[\mathbb{A}'], \mathbf{DC})$, *and* $S = Att_{\mathsf{T}'}(\mathcal{A}) \cup \{\lambda' \in \mathcal{T}_{\mathsf{T}'}(\mathcal{A}) \mid$ *for all* $\langle \mathcal{D}, \mathcal{C}, \lambda', \mathsf{T} \rangle \in \mathsf{cpd}(\lambda, \mathsf{T}), \mathsf{att\_free}(\mathcal{D}, \mathcal{C}, \lambda', \mathsf{T}') = false\}$

Set S is used to consider attacking lines along with every line that could result collaterally altered and turned into an attacking line. Thus, every attacking line is altered and from the alteration of the collateraly altered lines, future attacking lines are avoided to arise.

Observe that S (as well as any element of ATC) is calculated through a procedural process. To the contrary, ATC is defined to determine each of its elements purely from mathematical and logical principles. Hence, when the usage of functions like selection and padding is referred, it should not be confused to any kind of programming-like functions, *i.e.*, they are not invoking any procedure. Consequently, principles like preservation have the sole purpose of restricting the image of the selection function. Therefore, it is clear that only those active argumentation lines falling within the definition of the set S (see Def. 17) would be considered by the revision operation, and thereafter the image of the padding function (applied only to the selected arguments in lines from S) would be activated in the new theory.

Example 7 (Continues from Ex. 6)

In order to perform an AWR revision, a warranting padding is required. Therefore, we have that $Att_{\mathsf{T}'}(\mathcal{A}) = \{\lambda\}$ *and* $S = Att_{\mathsf{T}'}(\mathcal{A}) \cup \{\lambda_1\}$*, since* $\mathsf{att\_free}(\mathcal{C}_3, \mathcal{B}_3, \lambda_1, \mathsf{T}) = false\}$*. Following the consequent of the preservation principle,* $\gamma(\lambda_1) \in \lambda^{\uparrow}[\mathcal{B}_3]$ *must hold. The only option is to select* $\mathcal{B}_1$*, as* $\lambda_1^- = \{\mathcal{B}_1\}$*. Assuming a padded defeater* $\mathcal{D}_3$ *for* $\mathcal{B}_1$ *yields the tree on the right.*



The concluding results in this article assume a DAT $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsupseteq \rangle[\mathbb{A}], \mathbf{DC})$, an AWR Operator "$\times^{\omega}$", and an argument $\mathcal{A} \in \mathbb{U}$.

Lemma 3 *If* $\mathsf{T} \times^{\omega} \mathcal{A}$ *is performed through a warranting padding then* $Att_{(\mathsf{T} \times^{\omega} \mathcal{A})} = \emptyset$*.*

Proof sketch: *The fact that the revision is performed through a warranting padding ensures three things: first, every padding will be effective,* i.e., *it will actually activate the argument it intends to; second, it will comply with non-cumulativity, avoiding abnormal behaviors when side-effects interact in different lines in an undesired manner; third, at least preservation will be satisfied and every collateral effect will be handled properly. In addition to this, by definition, the revision operator alters every line that either was an attacking line from the original tree, or it was not, but collateral paddings would turn it into an attacking line (see set S in Def. 17). Hence, every attacking line will be properly altered, no matter if it was originally attacking or it had arisen from the revision process.*

Theorem 1 $\mathsf{T} \times^{\omega} \mathcal{A}$ *warrants* $\mathcal{A}$ iff $\mathsf{T} \times^{\omega} \mathcal{A}$ *is performed through a warranting padding.*

Proof: *Straightforward from Lemmas 1, 2 and 3.*

Corollary 1 $\mathsf{T} \times^{\omega} \mathcal{A} = \mathsf{T}$ iff *either* T *warrants* $\mathcal{A}$ *or* $\mathsf{T} \times^{\omega} \mathcal{A}$ *does not warrant* $\mathcal{A}$*.*

Lemma 3 ensures that revising through a warranting padding yields a theory with no attacking lines for the tree rooted in $\mathcal{A}$. Hence, under this condition, the revision is successful and warrants $\mathcal{A}$ (Theorem 1). Finally, Corollary 1 states that the revision does not change the original theory either when it already warranted $\mathcal{A}$ or the revision operation could not be successful, given that there was no warranting padding to perform it.

4 Worked Example: Legal Domain

In this section we will describe an actual legal case, in which a young man, Jack, is accused of killing his girlfriend, Lucy. This case is particularly interesting due to the large number of allegations posed at the trial –we, however, do not seek to imitate the actual course of events. In short, the prosecutor claims guilt by attempting to prove that the man had killed and burned the girl in a grill at his house, whereas the defense's case is that the girl died from a cocaine overdose and was afterwards burned by the man in a state of despair.

In order to make a practical example, we have extracted a smaller but representative amount of allegations, *i.e.*, arguments, and organized them within a dialectical tree. The tree's root is the initial argument posed by the defense, which ends up defeated, and then we will apply ATC's logical machinery to bring arguments that turn the root argument into a warrant. In this way, ATC proves to be a useful tool to perform hypothetical reasoning. We will assume a dialectical theory $\mathsf{T} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle [\mathbb{A}], \mathbf{DC})$, assuming that the set $\mathbf{DC}$ of dialectical constraints checks only for non-circularity, and the set $\mathbb{U}$ will include the following arguments:

$\mathcal{A}$: *Lucy died of cocaine overdose, so Jack did not kill her.*

$\mathcal{B}_1$: *Jack burned Lucy's remains; the burning objective was to hide the homicide.*

$\mathcal{B}_2$: *According to friends, Lucy never did drugs and she was afraid of cocaine.*

$\mathcal{B}_3$: *No traces of violence were found at the crime scene.*

$\mathcal{B}_4$: *There are no records of previous incidents between Jack and Lucy, so there is no motive for a homicide.*

$\mathcal{B}_5$: *An overdose might happen the first time a person consumes cocaine.*

$\mathcal{B}_6$: *Threads of hair belonging to Lucy were found nearby the crime scene implying violence.*

$\mathcal{B}_7$: *Psychological and psychiatric studies discovered Jack's psychopathic features.*

$\mathcal{D}_1$: *From the autopsy report, no cause of death could be determined.*

$\mathcal{D}_2$: *If the cause of death cannot be determined, there is no proof for a homicide.*

$\mathcal{D}_3$: *If the cause of death cannot be determined, there is no proof for an overdose.*

$\mathcal{D}_4$: *Recorded phone calls suggest that Lucy's friends were threatened to declare in her favor.*

We will assume that every argument is active, except for $\mathcal{D}_1$ and $\mathcal{D}_4$. Let also consider two more inactive arguments: $\mathcal{D}$ and $\mathcal{D}$', where:

- $\mathcal{D}_1 \sqsubseteq \mathcal{D}$ and $\mathcal{D}_2 \sqsubseteq \mathcal{D}$;
- $\mathcal{D}_1 \sqsubseteq \mathcal{D}'$ and $\mathcal{D}_3 \sqsubseteq \mathcal{D}'$.

The attack relation among active arguments is depicted in the tree in Fig. 3. Additionally, we have that $\mathcal{D} \hookrightarrow \mathcal{B}_1$, $\mathcal{D}' \hookrightarrow \mathcal{B}_5$ and $\mathcal{D}_4 \hookrightarrow \mathcal{B}_2$. Note that, in each argumentation line, arguments placed at odd positions are those posed by the defense, whereas the ones in even positions correspond to the prosecution.

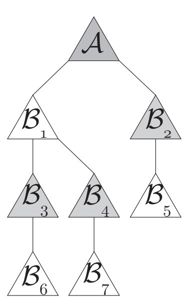


Figure 3: Dialectical tree defeating the defense's argument.

Now suppose the defense accounts with this information –representing the current state of the trial– and wishes to know how to proceed to change the outcome, *i.e.*, which allegations it should present. In order to warrant the defense's initial argument, at least one argument has to be activated in concordance with the approach defined in the present article. According to Def. 10, the set of attacking lines is $\{\lambda_1, \lambda_2\}$, with $\lambda_1 = [\mathcal{A}, \mathcal{B}_1, \mathcal{B}_3, \mathcal{B}_6]$ and $\lambda_2 = [\mathcal{A}, \mathcal{B}_1, \mathcal{B}_4, \mathcal{B}_7]$.

If we focus on λ_1, assuming that $\mathcal{B}_1 \prec_{\mathsf{T}} \mathcal{B}_6$, the selection there is $\mathcal{B}_1 \in \lambda_1^-$, and a defeater for $\mathcal{B}_1$ has to be activated. Since $\mathsf{idefs}(\mathcal{B}_1, \lambda_1, \mathsf{T}) \supseteq \{\mathcal{D}\}$ (see Def. 11), argument $\mathcal{B}_1$ is a suitable selection. The activation of $\mathcal{D}$ requires to activate $\mathcal{D}_1$ (provided that $\mathcal{D}_2$ is already active). Moreover, when activating $\mathcal{D}$, this argument itself turns out to be a collaterally padded defeater also for $\mathcal{B}_1$ but in line λ_2. Therefore, the *preservation* principle restricts the selection in λ_2 to be in $\lambda_2^{\uparrow}[\mathcal{B}_1]$. Note that $\gamma(\lambda_2)$ would equal $\gamma(\lambda_1)$. This happens due to the restriction imposed by preservation over the image of $\gamma(\lambda_2)$, *i.e.*, $\{\mathcal{A}, \mathcal{B}_1\}$. Afterwards, since such image is included in that of $\gamma(\lambda_1)$ (*i.e.*, $\{\mathcal{A}, \mathcal{B}_1, \mathcal{B}_3, \mathcal{B}_6\}$), the decision will be the same, as it is guided by the criterion "$\prec_{\mathsf{T}}$" among con arguments from the image. Hence, the activation of $\mathcal{D}$ turns both attacking lines into non-attacking, as depicted in Fig. 4.

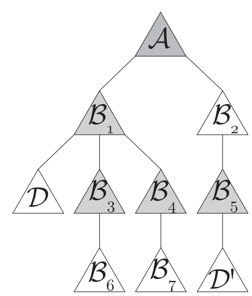


Figure 4: Activation of defeater $\mathcal{D}$ for $\mathcal{B}_1$

Nonetheless, the revision is not complete, since an undesirable side-effect has occurred: the activation of $\mathcal{D}_1$ has also activated argument $\mathcal{D}$', which attacks $\mathcal{B}_5$, since it dismisses $\mathcal{B}_5$'s overdose hypothesis. This collateral padding is also controlled by the *preservation* principle. Usually, the image of the selection function is restricted through the preservation principle, in this case the image remains the same given that the collateral padding arises as a new leaf in λ_3 and the upper segment leads to the same λ_3 –the image is effectively restricted only when the padding appears somewhere in the middle of a line. This is due to the nature of *preservation*, which is a generalized principle. Afterwards, given that the collateral alteration of λ_3 would turn the line into attacking, λ_3 verifies the definition of the set S in Def. 17. Therefore, λ_3 ends up being considered as if it was an attacking line, in order to avoid it to turn into the extended line $[\mathcal{A}, \mathcal{B}_2, \mathcal{B}_5, \mathcal{D}']$. Finally, argument $\mathcal{D}_4$ is activated, as depicted in Fig. 5. This figure shows that $\mathcal{A}$ ends up warranted from its revision in theory T. This operation is denoted as:

$$\mathsf{T} \times^{\omega} \mathcal{A} = (\langle \mathbb{U}, \hookrightarrow, \sqsubseteq \rangle [\mathbb{A} +^{\mathsf{T}} \{\mathcal{A}, \mathcal{D}_1, \mathcal{D}_4\}], \mathbf{DC})$$

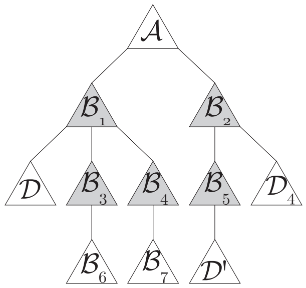


Figure 5: Dialectical tree for $\mathcal{A}$ resulting from $\mathsf{T} \times^{\omega} \mathcal{A}$

5 Conclusions & Related and Future Work

We have presented a complementary approach for argument revision considering activation of arguments. This new approach is comprehended within Argument Theory Change [Rotstein *et al.*, 2008b] and provides another standpoint to change the status of warrant of an argument. Moreover, it is the stepping stone towards the definition of a hybrid revision operator. This kind of operator would minimize change, improving the results achieved by any of the two single-approach operators.

Regarding related work, in [Cayrol *et al.*, 2008] the authors study change in the set of extensions of a system when an argument is added. However, they pose a strong restriction: the newly added argument must have at most one interaction (via attack) with an argument in the system. This restriction (which we do not assume) greatly simplifies the revision problem, as multiple interactions with the original system are difficult to handle. Moreover, we have the complexity added by subarguments, thus multiple arguments can arise from a single activation. Their objective differs from ours in that we focus on the warrant status of a single argument through the analysis of dialectical trees, whereas they study change over the set of extensions, by looking at an arguments graph. Other related articles [Moguillansky *et al.*, 2008; Rotstein *et al.*, 2008b] also work on ATC, and were discussed in the introduction.

Future work includes the definition of a hybrid approach for an argument revision operator. This method would decide between the activating and deactivating methods in order to choose the most profitable of both, in the sense of minimal change. It is important to note that this choice could have a different result in each attacking line; that is, an activation could be performed in one line while a deactivation is applied to another. To cope with this, a measure of change has to be assigned to each plausible activation/deactivation (*e.g.*, "activating argument $\mathcal{A}$ provokes less change than deactivating $\mathcal{B}$"). Since paddings require the existence of appropriate defeaters, sometimes there will be no choice to be made, and the hybrid approach will behave directly like the deactivating approach, thus ensuring success. Finally, the hybrid revision operator should provoke less or equal change than any of the other two individual approaches. Future work also includes the study of ATC purely within the area of belief revision and in particular its relation to the most popular model of change: AGM [Alchourrón *et al.*, 1985]. This will allow us to propose a set of rationality postulates accomodated to the argumentation theory, along with the representation theorems for each proposed approach of change in argumentation.

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